

KEK-TH-1505

# Structure of dimension-six derivative interactions in pseudo Nambu-Goldstone $N$ Higgs doublet models

---

Yohei Kikuta, Yasuhiro Okada and Yasuhiro Yamamoto

*KEK Theory Center, Institute of Particle and Nuclear Studies, KEK, 1-1 Oho,  
Tsukuba, Ibaraki 305-0801, Japan*

*Department of Particle and Nuclear Physics, Graduate University for Advanced Studies  
(Sokendai), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

*E-mail:* [kikuta@post.kek.jp](mailto:kikuta@post.kek.jp), [yasuhiro.okada@kek.jp](mailto:yasuhiro.okada@kek.jp),  
[yamayas@post.kek.jp](mailto:yamayas@post.kek.jp)

**ABSTRACT:** We derive the general structure of dimension-six derivative interactions in the  $N$  Higgs doublet models, where Higgs fields arise as pseudo Nambu-Goldstone modes of a strongly interacting sector. We show that there are several relations among the dimension-six operators, and therefore the number of independent operators decreases compared with models on which only  $SU(2)_L \times U(1)_Y$  invariance is imposed. As an explicit example, we derive scattering amplitudes and cross sections of longitudinal gauge bosons and Higgs bosons at high energy on models involving two Higgs doublets, and compare them with those of one Higgs doublet.

**KEYWORDS:** Beyond Standard Model, Higgs Physics, Technicolor and Composite Models.

---

## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>A brief review of the strongly interacting light Higgs model</b>	<b>2</b>
<b>3</b>	<b>Generalization to <math>N</math> Higgs doublet models</b>	<b>6</b>
3.1	Type I	7
3.2	Type II	8
3.3	Type III	10
3.4	Type IV	11
3.5	Type V	13
<b>4</b>	<b>Application to two Higgs doublet models</b>	<b>16</b>
4.1	Dimension-six derivative interactions	17
4.2	Scattering amplitudes of the longitudinal modes and the Higgs bosons	20
4.3	Cross sections and numerical results	24
<b>5</b>	<b>Conclusions</b>	<b>29</b>
<b>A</b>	<b>Generators of the <math>SO(4N)</math></b>	<b>30</b>
<b>B</b>	<b>Bidoublet notation</b>	<b>32</b>
<b>C</b>	<b>Amplitudes for 2HDM without the <math>SO(4)</math> symmetry</b>	<b>33</b>
<b>D</b>	<b>Elimination of the <math>\mathcal{O}^r</math> and <math>\mathcal{O}^{HT}</math></b>	<b>45</b>
<b>E</b>	<b>Cross sections of the central region</b>	<b>45</b>

---

## 1 Introduction

Recent progress in the Higgs boson searches at the Tevatron and the Large Hadron Collider (LHC) are remarkable [1], and we are likely to observe the Higgs boson soon if it exists in the mass range favored in the standard model (SM). When the Higgs boson is discovered, precise study of its properties is an important step to understand the dynamics behind the electroweak symmetry breaking (EWSB). In many models, a SM-like Higgs boson may be observed at first, even if the Higgs sector has complicated structure. For example, there may be a new symmetry principle like supersymmetry or a new strongly interacting sector like little Higgs models.

The strongly-interacting light Higgs (SILH) model was proposed as a framework describing an effective theory of one Higgs doublet models with a light physical Higgs boson

based on a new strongly interacting sector [2]. In this model, the Higgs doublet is identified as a composite field corresponding to pseudo Nambu-Goldstone bosons (PNGBs) of a global symmetry breaking at some high energy scale. The model introduces two new scales,  $f$  and  $m_\rho = g_\rho f$ , where  $f$  is the decay constant describing the global symmetry breaking and  $m_\rho$  is the mass scale of new resonances. The new coupling constant  $g_\rho$  is considered to be in the range of  $g_{SM} \lesssim g_\rho \lesssim 4\pi$ , where  $g_{SM}$  indicates the weak gauge coupling or the top Yukawa coupling. Explicit examples of this kind of structure can be found in little Higgs models [3] and models with large extra dimension [4]. In Ref. [2], the general form of the effective Lagrangian was introduced, and its phenomenological implications were discussed. They studied high energy behavior of the scattering amplitudes for longitudinal modes of massive gauge bosons and the Higgs boson. It was shown that the longitudinal gauge bosons and the physical Higgs boson production cross sections at the LHC satisfy a simple relation at high energy because these quantities are determined by the same dimension-six derivative coupling of the effective Lagrangian. This formulation was further investigated in details in Ref. [5].

In this paper, we generalize the SILH model to the model including  $N$  Higgs doublets. It is found that the number of independent dimension-six derivative interactions is strongly constrained by requiring that Higgs fields are generated as PNGBs of some global symmetry breaking. We derive the scattering amplitudes and cross sections of the longitudinal gauge bosons and the Higgs bosons at high energy for the two Higgs doublet model (2HDM). These scattering amplitudes are controlled by the dimension-six derivative interactions, and therefore study of these cross sections at the LHC and future Linear Colliders (LC) is important to identify this model experimentally.

This paper is organized as follows: In Sec. 2, we review the SILH model. In Sec. 3, we discuss the extension of the framework to the  $N$  Higgs doublet model (NHDM). Then we study phenomenological features of the 2HDM in Sec. 4. Section 5 is conclusion of our results.

## 2 A brief review of the strongly interacting light Higgs model

We briefly review the SILH model investigated in Ref. [2, 5].

In order to study scatterings of the longitudinal modes and the Higgs boson, we focus on dimension-six derivative interactions. The Lagrangian of the derivative interactions invariant under the  $SU(2)_L \times U(1)_Y$  symmetry is given by

$$\begin{aligned} \mathcal{L}^6 = & \frac{\lambda^H}{2\Lambda^2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H) + \frac{\lambda^r}{\Lambda^2} H^\dagger H (\partial^\mu H^\dagger) (\partial_\mu H) \\ & + \frac{\lambda^T}{2\Lambda^2} (H^\dagger \overleftrightarrow{\partial}_\mu H) (H^\dagger \overleftrightarrow{\partial}^\mu H) + i \frac{\lambda^{HT}}{\Lambda^2} \partial_\mu (H^\dagger H) (H^\dagger \overleftrightarrow{\partial}^\mu H) \end{aligned} \quad (2.1)$$

$$= \frac{\lambda^H}{2\Lambda^2} O^H + \frac{\lambda^r}{\Lambda^2} O^r + \frac{\lambda^T}{2\Lambda^2} O^T + i \frac{\lambda^{HT}}{\Lambda^2} O^{HT}, \quad (2.2)$$

where  $\lambda^H$ ,  $\lambda^r$ ,  $\lambda^T$  and  $\lambda^{HT}$  are real coefficients,  $H$  is an  $SU(2)_L$  doublet and  $H^\dagger \overleftrightarrow{\partial}_\mu H = H^\dagger (\partial_\mu H) - (\partial_\mu H)^\dagger H$ . We have introduced  $\Lambda$  as an appropriate cutoff scale. In this paper,

the SM gauge symmetry,  $SU(2)_L \times U(1)_Y$ , is treated as if it is global symmetry since we focus only scalar four point interactions with two derivatives. Full gauge invariant Lagrangian can be recovered by replacing a partial derivative with the covariant derivative. We study the effects of these derivative interactions with the requirement that the Higgs doublet is considered as a pseudo Nambu-Goldstone (NG) field.

In composite Higgs theories, the nonlinear realization [6] generates the derivative interactions like Eq. (2.2). For a nonlinear  $\sigma$  model (NL $\Sigma$ M) where a global symmetry is broken from  $G$  to  $H$ , the Lagrangian of NG fields is constructed with fields parametrizing the degenerate vacua of the quotient manifold  $G/H$ :

$$\xi = e^{i\Pi(x)/f}, \quad \Pi(x) = \Pi^a(x)X^a, \quad (2.3)$$

where the fields  $\Pi^a(x)$  represent the NG fields and  $\{X^a\}$  are generators of the broken symmetry  $G/H$ . Generators of the unbroken symmetry are denoted by  $\{T^i\}$ . Commutation relations among these generators are

$$[T^i, T^j] = if^{ijk}T^k, \quad [T^i, X^a] = if^{iab}X^b, \quad [X^a, X^b] = if^{abi}T^i + if^{abc}X^c. \quad (2.4)$$

If the second term of the right hand side vanishes for the third commutator ( $f^{abc} = 0$ ),  $G/H$  is called symmetric space. In order to construct the nonlinear realization, the Maurer-Cartan one form,  $\alpha_\mu(\Pi)$ , is a fundamental constituent:

$$\alpha_\mu(\Pi) = -i\xi^{-1}(\Pi) \partial_\mu \xi(\Pi) \quad (2.5)$$

$$= \frac{1}{f} \partial_\mu \Pi - \frac{i}{2f^2} [\Pi, \partial_\mu \Pi] - \frac{1}{6f^3} [\Pi, [\Pi, \partial_\mu \Pi]] + \mathcal{O}((\Pi/f)^4) \quad (2.6)$$

$$= \alpha_{\perp\mu}^a(\Pi) X^a + \alpha_{\parallel\mu}^i(\Pi) T^i \quad (2.7)$$

$$= \alpha_{\perp\mu} + \alpha_{\parallel\mu}. \quad (2.8)$$

Under a transformation  $g (\in G)$ , we obtain

$$\alpha_{\perp\mu}(\Pi) \rightarrow \alpha_{\perp\mu}(\Pi') = h(\Pi, g) \alpha_{\perp\mu}(\Pi) h^{-1}(\Pi, g), \quad (2.9)$$

$$\alpha_{\parallel\mu}(\Pi) \rightarrow \alpha_{\parallel\mu}(\Pi') = h(\Pi, g) \alpha_{\parallel\mu}(\Pi) h^{-1}(\Pi, g) - ih(\Pi, g) \partial_\mu h^{-1}(\Pi, g), \quad (2.10)$$

where  $h \in H$ . Using Eqs. (2.4) and (2.9), it is straightforward to calculate  $\alpha_{\perp\mu}$ :

$$\begin{aligned} \alpha_{\perp\mu}(\Pi) &= \frac{1}{f} \partial_\mu \Pi - \frac{i}{2f^2} [\Pi, \partial_\mu \Pi]_X - \frac{1}{6f^3} [\Pi, [\Pi, \partial_\mu \Pi]]_X + \mathcal{O}((\Pi/f)^4) \\ &= X^a \left( \frac{1}{f} \partial_\mu \Pi^a + \frac{1}{2f^2} f^{abc} \Pi^b \partial_\mu \Pi^c + \frac{1}{6f^3} (f^{abe} f^{cde} + f^{abi} f^{cdi}) \Pi^b \Pi^c \partial_\mu \Pi^c \right) + \mathcal{O}((\Pi/f)^4), \end{aligned} \quad (2.11)$$

where  $[X^a, X^b]_X$  is the projection of the commutator into the broken generator. Then the  $G$ -invariant two-derivative term of the NG fields is given by

$$\begin{aligned} \frac{f^2}{2} \text{Tr}[\alpha_{\perp\mu} \alpha_{\perp}^\mu] &= \frac{1}{2} (\partial_\mu \Pi^a) (\partial^\mu \Pi^a) \\ &\quad - \left( \frac{1}{6f^2} f^{aci} f^{bdi} + \frac{1}{24f^2} f^{ace} f^{bde} \right) \Pi^a \Pi^b (\partial_\mu \Pi^c) (\partial^\mu \Pi^d) + \mathcal{O}((\Pi/f)^4). \end{aligned} \quad (2.12)$$

The normalizations of generators are given by  $\text{Tr}[T^i T^j] = \delta^{ij}$  and  $\text{Tr}[X^a X^b] = \delta^{ab}$ .

We assume that the  $SO(4)$  multiplet,  $h^a$  ( $a \in \{1, \dots, 4\}$ ), is embedded in the NG fields,  $\Pi^a$ . This multiplet corresponds to the  $SU(2)$  doublet Higgs field. In this paper, we treat the Higgs doublet as a NG field and ignore possible other fields. The second term of Eq. (2.12) leads to dimension-six derivative interactions of the Higgs fields. We write the derivative interaction as

$$\mathcal{L}^{6\text{NL}} = \frac{1}{f^2} \mathcal{T}_{abcd} h^a h^b \partial_\mu h^c \partial^\mu h^d, \quad (2.13)$$

$$\mathcal{T}_{abcd} = -\frac{1}{6} f^{aci} f^{bdi} - \frac{1}{24} f^{ace} f^{bde}. \quad (2.14)$$

Since the structure constant is totally antisymmetric,  $f^{aci}$  is a  $4 \times 4$  antisymmetric matrix for each  $i$ . Since the Lagrangian must be the  $SU(2)_L \times U(1)_Y$  invariant, it is useful to use the generators of  $SO(4) \simeq SU(2)_L \times SU(2)_R$  as a complete set of antisymmetric matrices. Our definitions of the generators,  $T^{L\alpha}$  and  $T^{R\beta}$  ( $\alpha, \beta \in \{1, 2, 3\}$ ), are listed in Appendix A. In particular, the hyper charge generator corresponds to  $T^{R3}$ .

We parametrize the  $SU(2)_L$  Higgs doublet as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} h^1 + i h^2 \\ h^3 + i h^4 \end{pmatrix}. \quad (2.15)$$

In order to see the property of the  $SU(2)_R$  symmetry, it is useful to use the bidoublet notation:

$$\Phi = (i\sigma^2 H^* H) \quad (2.16)$$

$$= \frac{1}{\sqrt{2}} (\mathbf{1}_2 h^3 + i(\sigma^1 h^2 + \sigma^2 h^1 - \sigma^3 h^4)), \quad (2.17)$$

where  $\mathbf{1}_2$  is the  $2 \times 2$  unit matrix. With the bidoublet,  $SO(4)$  transformation for  $h^a$  is represented as

$$\Phi \rightarrow L \Phi R^\dagger, \quad (2.18)$$

where  $L \in SU(2)_L$  and  $R \in SU(2)_R$ . We can show the following relations to connect the  $SO(4)$  multiplet with the bidoublet:

$$h^a (T^{L\alpha})_{ac} \partial_\mu h^c = \frac{1}{4} \text{Tr} \left[ \Phi^\dagger \sigma^\alpha \overleftrightarrow{\partial}_\mu \Phi \right], \quad (2.19)$$

$$h^a (T^{R\beta})_{ac} \partial_\mu h^c = \frac{1}{4} \text{Tr} \left[ \Phi \sigma^\beta \overleftrightarrow{\partial}_\mu \Phi^\dagger \right]. \quad (2.20)$$

Hence, the following form of  $\mathcal{T}_{abcd}$  is invariant under the  $SU(2)_L \times U(1)_Y$  symmetry:

$$\mathcal{T}_{abcd} = a^L (T^{L\alpha})_{ac} (T^{L\alpha})_{bd} + a^R (T^{R\beta})_{ac} (T^{R\beta})_{bd} + a^Y (T^{R3})_{ac} (T^{R3})_{bd}. \quad (2.21)$$

With the explicit representation given in Appendix A, the tensor,  $\mathcal{T}_{abcd}$ , can be written as

$$\begin{aligned} \mathcal{T}_{abcd} = & -\frac{a^L + a^R}{4} (\delta^{ab} \delta^{cd} - \delta^{ad} \delta^{bc}) + \frac{a^L - a^R}{4} \epsilon_{abcd} \\ & - \frac{a^Y}{4} (\delta^{1a} \delta^{2c} - \delta^{1c} \delta^{2a} + \delta^{3a} \delta^{4c} - \delta^{3c} \delta^{4a}) (\delta^{1b} \delta^{2d} - \delta^{1d} \delta^{2b} + \delta^{3b} \delta^{4d} - \delta^{3d} \delta^{4b}). \end{aligned} \quad (2.22)$$

According to Eq. (2.13), the second term of the above equation does not contribute to the Lagrangian due to the Bose symmetry. This means that degree of freedom (DOF) is decreased by the NLSM structure. After expanding it in terms of  $O^H$ ,  $O^r$ ,  $O^T$  and  $O^{HT}$ , the following Lagrangian is obtained:

$$\mathcal{L}^{6\text{NL}} = \frac{a^L + a^R}{4f^2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H) - \frac{a^L + a^R}{f^2} H^\dagger H (\partial_\mu H^\dagger) (\partial^\mu H) + \frac{a^Y}{4f^2} (H^\dagger \overleftrightarrow{\partial}_\mu H) (H^\dagger \overleftrightarrow{\partial}^\mu H). \quad (2.23)$$

In the SILH model, the number of independent coefficients is two while there are four DOF in the Lagrangian preserving only the  $SU(2)_L \times U(1)_Y$  symmetry.

In the rest of this section, the scattering amplitudes of the longitudinal gauge bosons and the Higgs boson are studied as phenomenological consequences of the Lagrangian. The given Lagrangian (2.23) can be simplified with the field redefinition:

$$H^a \rightarrow H^a + \frac{a}{f^2} H^a (H^\dagger H). \quad (2.24)$$

To study the scatterings of gauge boson longitudinal modes, it is enough to see the effect of the redefinition in the kinetic term:

$$(\partial_\mu H)^\dagger (\partial^\mu H) \rightarrow (\partial_\mu H)^\dagger (\partial^\mu H) + \frac{a}{f^2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H) + \frac{2a}{f^2} H^\dagger H (\partial_\mu H^\dagger) (\partial^\mu H) + \mathcal{O}((H/f)^4). \quad (2.25)$$

This indicates that we can always choose the coefficient,  $a$ , so as to eliminate  $O^r$ . The Yukawa interaction and the Higgs potential are also changed by the redefinition. However, these  $\mathcal{O}(v^2/f^2)$  corrections can be neglected in leading contributions of the derivative interactions. After the redefinition, the Lagrangian becomes

$$\mathcal{L}^{6\text{NL}} = \frac{3(a^L + a^R)}{4f^2} O^H + \frac{a^Y}{4f^2} O^T. \quad (2.26)$$

The term  $O^T$  breaks the custodial symmetry so that the coefficient  $a^Y$  is severely constrained by electroweak precision measurements. The term  $O^H$  affects high energy behavior in the scattering amplitudes of the longitudinal modes and the Higgs boson. If we keep only  $O^H$ , using the equivalence theorem [7], these amplitudes in high energy region ( $s, t, u \gg m_h^2, m_W^2$  and  $s + t + u = 0$ ) are given as follows:

$$\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \frac{3(s+t)}{2f^2} (a^L + a^R), \quad (2.27)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L Z_L) = \mathcal{M}(W_L^+ W_L^- \rightarrow hh) = \frac{3s}{2f^2} (a^L + a^R), \quad (2.28)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L h) = 0. \quad (2.29)$$

In the SILH model, the amplitudes increase as  $E^2$  in high energy region whereas such behavior does not appear in the SM case ( $f \rightarrow \infty$ ). The amplitudes are supposed to grow up to the scale,  $m_\rho$ , where new resonances are produced. It is, therefore, important to precisely measure vector boson fusion processes in high energy region and see correlations among them at the LHC and LC. Studies of the scatterings at the LHC are done in Refs. [8–10]. Above the scale  $m_\rho$ , we need to include the effects of heavy resonances, see, e.g. [11].

### 3 Generalization to $N$ Higgs doublet models

In this section, we present the generalizations of the SILH model which includes only one Higgs doublet to the case with  $N$  Higgs doublets.

The  $SU(2)_L \times U(1)_Y$  invariant Lagrangian for dimension-six derivative interactions consists of four kinds of operators:

$$\mathcal{O}^H : O_{ijkl}^H = \partial_\mu (H_i^\dagger H_j) \partial^\mu (H_k^\dagger H_l), \quad (3.1)$$

$$\mathcal{O}^T : O_{ijkl}^T = (H_i^\dagger \overleftrightarrow{\partial}_\mu H_j) (H_k^\dagger \overleftrightarrow{\partial}^\mu H_l), \quad (3.2)$$

$$\mathcal{O}^r : O_{ijkl}^r = H_i^\dagger H_j (\partial_\mu H_k)^\dagger (\partial^\mu H_l), \quad (3.3)$$

$$\mathcal{O}^{HT} : O_{ijkl}^{HT} = \partial_\mu (H_i^\dagger H_j) (H_k^\dagger \overleftrightarrow{\partial}^\mu H_l), \quad (3.4)$$

where  $i, j, k, l \in \{1, \dots, N\}$  stand for the species of Higgs doublets.

In order to study how the nonlinear nature of the Higgs sector reduces the number of independent coefficients, we classify above operators into the following five types:

**Type I:** All species are same (e.g.  $\partial_\mu (H_i^\dagger H_i) \partial^\mu (H_i^\dagger H_i)$ ).

**Type II:** Only one species is different from the others (e.g.  $\partial_\mu (H_i^\dagger H_i) \partial^\mu (H_i^\dagger H_j)$ ).

**Type III:** Two pairs of different species appear (e.g.  $\partial_\mu (H_i^\dagger H_i) \partial^\mu (H_j^\dagger H_j)$ ).

**Type IV:** Three species are included (e.g.  $\partial_\mu (H_i^\dagger H_i) \partial^\mu (H_j^\dagger H_k)$ ).

**Type V:** All species are deferent (e.g.  $\partial_\mu (H_i^\dagger H_j) \partial^\mu (H_k^\dagger H_l)$ ).

On the other hand, dimension-six derivative interactions of the NG fields are written as Eq. (2.12). In this section, we regard the NG fields,  $\Pi^a$ , as not an  $SO(4)$  multiplet but an  $SO(4N)$  multiplet,  $h^a$  ( $a \in \{1, \dots, 4N\}$ ), and possible other NG fields are neglected. In this case, the structure constant,  $f^{aci}$ , in Eq. (2.14) is considered as  $4N \times 4N$  antisymmetric matrices for given  $i$ . Namely it can be expressed using generators of the  $SO(4N)$ . The multiplet corresponds to  $N$  species of Higgs doublets:

$$H_i = \frac{1}{\sqrt{2}} \begin{pmatrix} h^{1+4(i-1)} + ih^{2+4(i-1)} \\ h^{3+4(i-1)} + ih^{4+4(i-1)} \end{pmatrix}, \quad (3.5)$$

where  $i$  ( $i = 1, \dots, N$ ) is also the index of the Higgs species.

To see how the  $SU(2)_L \times U(1)_Y$  symmetry is imposed on the theory, it is convenient to use the bidoublet notation like Eq. (2.16). In the case of  $N = 1$ , generators which correspond to  $SU(2)_L$ ,  $T^{L\alpha}$ , and  $SU(2)_R$ ,  $T^{R\beta}$ , can be used as generators of the  $SO(4)$ . See Eqs. (2.19) and (2.20). They are the  $(\mathbf{3}, \mathbf{1})$  and  $(\mathbf{1}, \mathbf{3})$  representations of  $SU(2)_L \times SU(2)_R$ . In the case of  $N \geq 2$ , other kind of generator appears, namely, the  $(\mathbf{1}, \mathbf{1})$  and  $(\mathbf{3}, \mathbf{3})$  representations of the  $SU(2)_L \times SU(2)_R$  written as  $U$  and  $S^{\alpha\beta}$ , respectively. Explicit forms of them are shown in Appendix A. With the generators, the following relations are

obtained between the multiplet of the  $SO(4N)$  and the bidoublet:

$$h^a \left( T_{(i,i)}^{L\alpha} \right)_{ac} \partial_\mu h^c = \frac{1}{4} \text{Tr} \left[ \Phi_{ii}^\dagger \sigma^\alpha \overleftrightarrow{\partial}_\mu \Phi_{ii} \right], \quad (3.6)$$

$$h^a \left( T_{(i,i)}^{R\beta} \right)_{ac} \partial_\mu h^c = \frac{1}{4} \text{Tr} \left[ \Phi_{ii} \sigma^\beta \overleftrightarrow{\partial}_\mu \Phi_{ii}^\dagger \right], \quad (3.7)$$

$$h^a \left( T_{(i,j)}^{L\alpha} \right)_{ac} \partial_\mu h^c = \frac{1}{4} \text{Tr} \left[ \Phi_{ii}^\dagger \sigma^\alpha \overleftrightarrow{\partial}_\mu \Phi_{jj} + \Phi_{jj}^\dagger \sigma^\alpha \overleftrightarrow{\partial}_\mu \Phi_{ii} \right], \quad (3.8)$$

$$h^a \left( T_{(i,j)}^{R\beta} \right)_{ac} \partial_\mu h^c = \frac{1}{4} \text{Tr} \left[ \Phi_{ii} \sigma^\beta \overleftrightarrow{\partial}_\mu \Phi_{jj}^\dagger + \Phi_{jj} \sigma^\beta \overleftrightarrow{\partial}_\mu \Phi_{ii}^\dagger \right], \quad (3.9)$$

$$h^a \left( U_{(i,j)} \right)_{ac} \partial_\mu h^c = \frac{i}{4} \text{Tr} \left[ \Phi_{jj}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{ii} + \Phi_{jj} \overleftrightarrow{\partial}_\mu \Phi_{ii}^\dagger \right], \quad (3.10)$$

$$h^a \left( S_{(i,j)}^{\alpha\beta} \right)_{ac} \partial_\mu h^c = \frac{i}{4} \text{Tr} \left[ \Phi_{ii}^\dagger \sigma^\alpha \overleftrightarrow{\partial}_\mu \Phi_{jj} \sigma^\beta + \Phi_{ii} \sigma^\beta \overleftrightarrow{\partial}_\mu \Phi_{jj}^\dagger \sigma^\alpha \right], \quad (3.11)$$

where

$$\Phi_{ii} = (i\sigma^2 H_i^* H_i). \quad (3.12)$$

The  $SU(2)_L \times SU(2)_R$  transformation is defined by

$$\Phi_{ii} \rightarrow L \Phi_{ii} R^\dagger. \quad (3.13)$$

If the Higgs sector is invariant under the transformation (3.13), it preserves the  $SO(4)$  symmetry which is directly connected with the custodial symmetry. We simply refer the  $SO(4)$  invariance as the custodial invariance. For details of the bidoublet notation. See Appendix B.

The  $SU(2)_L \times U(1)_Y$  invariant forms can be constructed in two ways. One is to contract the indices of  $SU(2)_L$  or  $SU(2)_R$ , which gives  $SO(4)$  invariant terms. The other is to keep only  $\sigma^3$  term of  $T^{R3}$ , which corresponds to  $SO(4)$  breaking terms<sup>1</sup>.

In the following subsections, we show that the number of independent derivative interactions needed to describe  $N$  Higgs doublets in the NL $\Sigma$ M is reduced compared to that of all operators invariant under  $SU(2)_L \times U(1)_Y$ .

### 3.1 Type I

The Lagrangian for type I can be constructed in the same way as the SILH model reviewed in the previous section. In order to show our notation for the NHDM, we rewrite several equations.

The Lagrangian of derivative interactions invariant under  $SU(2)_L \times U(1)_Y$  is

$$\mathcal{L}_I^6 = \frac{\lambda_{iii}^H}{2\Lambda^2} O_{iii}^H + \frac{\lambda_{iii}^r}{\Lambda^2} O_{iii}^r + \frac{\lambda_{iii}^T}{2\Lambda^2} O_{iii}^T + i \frac{\lambda_{iii}^{HT}}{\Lambda^2} O_{iii}^{HT}, \quad (3.14)$$

where all of the coefficients,  $\lambda^H$ ,  $\lambda^r$ ,  $\lambda^T$  and  $\lambda^{HT}$ , are real so that there are  $3N$  real and  $N$  imaginary DOF in the general NHDM.

---

<sup>1</sup> There is a special case where the  $SO(4)$  invariance is preserved with combination of  $T^{R3}$  terms. See Sec. 4.2.



For the case of the NL $\Sigma$ M, as is given in Eq. (2.13), the Lagrangian corresponding to the above can be parametrized as

$$\mathcal{L}_I^{6\text{NL}} = \frac{1}{f^2} \mathcal{T}_{abcd}^I h^a h^b (\partial_\mu h^c) (\partial^\mu h^d), \quad (3.15)$$

where

$$\mathcal{T}_{abcd}^I = a_{iiii}^L \left( T_{(i,i)}^{L\alpha} \right)_{ac} \left( T_{(i,i)}^{L\alpha} \right)_{bd} + a_{iiii}^R \left( T_{(i,i)}^{R\beta} \right)_{ac} \left( T_{(i,i)}^{R\beta} \right)_{bd} + 2a_{iiii}^Y \left( T_{(i,i)}^{R3} \right)_{ac} \left( T_{(i,i)}^{R3} \right)_{bd}. \quad (3.16)$$

The coefficients,  $a_{iiii}^L$ ,  $a_{iiii}^R$  and  $a_{iiii}^Y$  are real. Substituting Eq. (3.16) into Eq. (3.15), the following Lagrangian is obtained:

$$\mathcal{L}_I^{6\text{NL}} = \frac{c_{[i]}^{I1}}{4f^2} O_{iiii}^H - \frac{c_{[i]}^{I1}}{f^2} O_{iiii}^r + \frac{c_{[i]}^{I2}}{2f^2} O_{iiii}^T, \quad (3.17)$$

$$c_{[i]}^{I1} = a_{iiii}^L + a_{iiii}^R, \quad (3.18)$$

$$c_{[i]}^{I2} = a_{iiii}^Y. \quad (3.19)$$

Note that the factor of 2 in Eq. (3.16) is introduced; this is different from Eq. (2.21).

In NHDM, there are  $3N$  real and  $N$  imaginary DOF. However, assuming the Higgs doublets are embedded in the NL $\Sigma$ M,  $2N$  real DOF remain (Table 1). For example, in the case of  $N = 1$ , three real and one imaginary DOF decrease to two real DOF. It is consistent with the result in the previous section.

	Re	Im
General	$3N$	$N$
$\mathcal{O}^H$	$N$	$0$
$\mathcal{O}^T$	$N$	$0$
$\mathcal{O}^r$	$N$	$0$
$\mathcal{O}^{HT}$	$0$	$N$
Nonlinear	$2N$	$0$

**Table 1.** The number of the independent dimension-six derivative interactions for type I. The first and second columns mean real and imaginary DOF, respectively. We show the number of DOF for the case where  $SU(2)_L \times U(1)_Y$  invariance is imposed (General) and the case with nonlinear realization (Nonlinear). In the case of General, the DOF of each type of operators is also shown.

### 3.2 Type II

For type II, the Lagrangian of the dimension-six derivative interactions is

$$\mathcal{L}_{\text{II}}^6 = \frac{\lambda_{iiij}^H}{\Lambda^2} O_{iiij}^H + \frac{\lambda_{iiij}^r}{\Lambda^2} O_{iiij}^r + \frac{\lambda_{ijii}^r}{\Lambda^2} O_{ijii}^r + \frac{\lambda_{iiij}^T}{\Lambda^2} O_{iiij}^T + \frac{\lambda_{iiij}^{HT}}{\Lambda^2} O_{iiij}^{HT} + \frac{\lambda_{ijii}^{HT}}{\Lambda^2} O_{ijii}^{HT} + \text{H.c.} \quad (3.20)$$

All of the coefficients can be complex so that there are  $6N(N-1)$  real and imaginary DOF, respectively.

Assuming the Higgs doublets are described as the NG fields, the Lagrangian can be written as

$$\mathcal{L}_{\text{II}}^{6\text{NL}} = \frac{1}{f^2} \mathcal{T}_{abcd}^{\text{II}} h^a h^b (\partial_\mu h^c) (\partial^\mu h^d), \quad (3.21)$$

where, using real coefficients,

$$\begin{aligned} \mathcal{T}_{abcd}^{\text{II}} = & 2a_{iii}^L \left( T_{(i,i)}^{L\alpha} \right)_{ac} \left( T_{(i,j)}^{L\alpha} \right)_{bd} + 2a_{iii}^R \left( T_{(i,i)}^{R\beta} \right)_{ac} \left( T_{(i,j)}^{R\beta} \right)_{bd} + 2a_{iii}^{LS} \left( T_{(i,i)}^{L\alpha} \right)_{ac} \left( S_{(i,j)}^{\alpha 3} \right)_{bd} \\ & + 4a_{iii}^Y \left( T_{(i,i)}^{R3} \right)_{ac} \left( T_{(i,j)}^{R3} \right)_{bd} + 4a_{iii}^{YU} \left( T_{(i,i)}^{R3} \right)_{ac} \left( U_{(i,j)} \right)_{bd}. \end{aligned} \quad (3.22)$$

The Lagrangian,  $\mathcal{L}_{\text{II}}^{6\text{NL}}$ , can be expanded in terms of the  $SU(2)_L$  doublets,  $H_i$ . The normalization of the coefficients are chosen to make the following coefficients simple:

$$\mathcal{L}_{\text{II}}^{6\text{NL}} = \frac{c_{[ij]}^{\text{II1}}}{2f^2} \mathcal{O}_{iii}^H - \frac{c_{[ij]}^{\text{II1}}}{f^2} \mathcal{O}_{iii}^r - \frac{c_{[ij]}^{\text{II1}}}{f^2} \mathcal{O}_{iji}^r + \frac{c_{[ij]}^{\text{II2}}}{f^2} \mathcal{O}_{iii}^T + \text{H.c.}, \quad (3.23)$$

$$c_{[ij]}^{\text{II1}} = a_{iii}^L + a_{iii}^R - ia_{iii}^{LS}, \quad (3.24)$$

$$c_{[ij]}^{\text{II2}} = a_{iii}^Y - ia_{iii}^{YU}. \quad (3.25)$$

The original Lagrangian,  $\mathcal{L}_{\text{II}}^6$ , has  $6N(N-1)$  real and imaginary DOF. However, the corresponding Lagrangian consists of  $2N(N-1)$  real and imaginary DOF, respectively, if the Higgs doublets are considered as NG fields (Table 2).

	Re	Im
General	$6N(N-1)$	$6N(N-1)$
$\mathcal{O}^H$	$N(N-1)$	$N(N-1)$
$\mathcal{O}^T$	$N(N-1)$	$N(N-1)$
$\mathcal{O}^r$	$2N(N-1)$	$2N(N-1)$
$\mathcal{O}^{HT}$	$2N(N-1)$	$2N(N-1)$
Nonlinear	$2N(N-1)$	$2N(N-1)$

**Table 2.** The number of the independent dimension-six derivative interactions for type II. The first and second columns mean real and imaginary DOF, respectively. We show the number of DOF for the case where  $SU(2)_L \times U(1)_Y$  invariance is imposed (General) and the case with nonlinear realization (Nonlinear). In the case of General, the DOF of each type of operators is also shown.

### 3.3 Type III

The Lagrangian of type III derivative interactions is

$$\begin{aligned}
\mathcal{L}_{\text{III}}^6 = & \frac{\lambda_{iijj}^H}{2\Lambda^2} O_{iijj}^H + \frac{\lambda_{ijji}^H}{2\Lambda^2} O_{ijji}^H + \frac{\lambda_{ijij}^H}{2\Lambda^2} O_{ijij}^H \\
& + \frac{\lambda_{iijj}^r}{2\Lambda^2} O_{iijj}^r + \frac{\lambda_{jjii}^r}{2\Lambda^2} O_{jjii}^r + \frac{\lambda_{ijij}^r}{\Lambda^2} O_{ijij}^r + \frac{\lambda_{ijji}^r}{\Lambda^2} O_{ijji}^r \\
& + \frac{\lambda_{iijj}^T}{2\Lambda^2} O_{iijj}^T + \frac{\lambda_{ijji}^T}{2\Lambda^2} O_{ijji}^T + \frac{\lambda_{ijij}^T}{2\Lambda^2} O_{ijij}^T \\
& + i \frac{\lambda_{iijj}^{HT}}{2\Lambda^2} O_{iijj}^{HT} + i \frac{\lambda_{jjii}^{HT}}{2\Lambda^2} O_{jjii}^{HT} + \frac{\lambda_{ijij}^{HT}}{\Lambda^2} O_{ijij}^{HT} + \frac{\lambda_{ijji}^{HT}}{\Lambda^2} O_{ijji}^{HT} + \text{H.c.} .
\end{aligned} \tag{3.26}$$

In the above operators,  $O_{iijj}^H$ ,  $O_{ijji}^H$ ,  $O_{iijj}^r$ ,  $O_{jjii}^r$ ,  $O_{iijj}^T$  and  $O_{ijji}^T$ , are Hermitian operators while  $O_{iijj}^{HT}$  and  $O_{ijji}^{HT}$  are skew Hermitian operators. The number of independent real and imaginary coefficients are  $6N(N-1)$  and  $4N(N-1)$ , respectively.

If the Higgs doublets are embedded in the NG fields, Lagrangian can be written as follows:

$$\mathcal{L}_{\text{III}}^{6\text{NL}} = \frac{1}{f^2} \mathcal{T}_{abcd}^{\text{III}} h^a h^b (\partial_\mu h^c) (\partial^\mu h^d). \tag{3.27}$$

Parametrization of the tensor,  $\mathcal{T}_{abcd}^{\text{III}}$ , is

$$\begin{aligned}
\mathcal{T}_{abcd}^{\text{III}} = & 2a_{iijj}^L (T_{(i,i)}^{L\alpha})_{ac} (T_{(j,j)}^{L\alpha})_{bd} + a_{ijij}^L (T_{(i,j)}^{L\alpha})_{ac} (T_{(i,j)}^{L\alpha})_{bd} + 2a_{iijj}^R (T_{(i,i)}^{R\beta})_{ac} (T_{(j,j)}^{R\beta})_{bd} \\
& + a_{ijij}^R (T_{(i,j)}^{R\beta})_{ac} (T_{(i,j)}^{R\beta})_{bd} + a_{iijj}^S (S_{(i,j)}^{\alpha 3})_{ac} (S_{(i,j)}^{\alpha 3})_{bd} + a_{ijij}^{SS} (S_{(i,j)}^{\alpha\beta})_{ac} (S_{(i,j)}^{\alpha\beta})_{bd} \\
& + a_{ijij}^{LS} (T_{(i,j)}^{L\alpha})_{ac} (S_{(i,j)}^{\alpha 3})_{bd} + 2a_{iijj}^Y (T_{(i,i)}^{R3})_{ac} (T_{(j,j)}^{R3})_{bd} + a_{ijij}^Y (T_{(i,j)}^{R3})_{ac} (T_{(i,j)}^{R3})_{bd} \\
& + a_{ijij}^U (U_{(i,j)})_{ac} (U_{(i,j)})_{bd} + a_{ijij}^{YU} (T_{(i,j)}^{R3})_{ac} (U_{(i,j)})_{bd},
\end{aligned} \tag{3.28}$$

where the coefficients are real. The derivative interactions embedded in NL $\Sigma$ M can be written as

$$\begin{aligned}
\mathcal{L}_{\text{III}}^{6\text{NL}} = & \frac{c_{[ij]}^{\text{III1}}}{2f^2} O_{iijj}^H + \frac{c_{[ij]}^{\text{III2}}}{2f^2} O_{ijji}^H - \frac{c_{[ij]}^{\text{III1}}}{f^2} (O_{iijj}^r + O_{jjii}^r) \\
& - \frac{c_{[ij]}^{\text{III2}}}{f^2} (O_{ijji}^r + O_{iijj}^r) + \frac{c_{[ij]}^{\text{III3}}}{2f^2} O_{ijji}^T + \frac{c_{[ij]}^{\text{III4}}}{2f^2} O_{iijj}^T \\
& + \left( \frac{c_{[ij]}^{\text{III5}}}{4f^2} O_{ijij}^H - \frac{c_{[ij]}^{\text{III5}}}{f^2} O_{ijij}^r + \frac{c_{[ij]}^{\text{III6}}}{4f^2} O_{ijij}^T + \text{H.c.} \right),
\end{aligned} \tag{3.29}$$

$$c_{[ij]}^{\text{III1}} = a_{ijij}^L + a_{ijij}^R + a_{ijij}^S + 2a_{ijij}^{SS}, \tag{3.30}$$

$$c_{[ij]}^{\text{III2}} = a_{iijj}^L + a_{ijij}^R - a_{ijij}^{SS}, \tag{3.31}$$

$$c_{[ij]}^{\text{III3}} = -a_{ijij}^L + a_{iijj}^L - a_{ijij}^S + a_{ijij}^Y + a_{ijij}^U, \tag{3.32}$$

$$c_{[ij]}^{\text{III4}} = -a_{iijj}^L + a_{ijij}^L - a_{ijij}^R + a_{ijij}^R + a_{ijij}^S + a_{ijij}^Y, \tag{3.33}$$

$$c_{[ij]}^{\text{III5}} = a_{ijij}^L + a_{iijj}^R - a_{ijij}^S - a_{ijij}^{SS} - ia_{ijij}^{LS}, \tag{3.34}$$

$$c_{[ij]}^{\text{III6}} = a_{ijij}^R - a_{iijj}^R + a_{ijij}^Y - a_{ijij}^U - ia_{ijij}^{YU}. \tag{3.35}$$

Because of the nature of the nonlinear dynamics, the number of independent DOF decreases from  $6N(N-1)$  real DOF and  $4N(N-1)$  imaginary DOF to  $3N(N-1)$  real DOF and  $N(N-1)$  imaginary DOF (Table 3).

	Re	Im
General	$6N(N-1)$	$4N(N-1)$
$\mathcal{O}^H$	$(3/2)N(N-1)$	$(1/2)N(N-1)$
$\mathcal{O}^T$	$(3/2)N(N-1)$	$(1/2)N(N-1)$
$\mathcal{O}^r$	$2N(N-1)$	$N(N-1)$
$\mathcal{O}^{HT}$	$N(N-1)$	$2N(N-1)$
Nonlinear	$3N(N-1)$	$N(N-1)$

**Table 3.** The number of the independent dimension-six derivative interactions for type III. The first and second columns mean real and imaginary DOF, respectively. We show the number of DOF for the case where  $SU(2)_L \times U(1)_Y$  invariance is imposed (General) and the case with nonlinear realization (Nonlinear). In the case of General, the DOF of each type of operators is also shown.

### 3.4 Type IV

The Lagrangian of type IV derivative interactions is

$$\begin{aligned}
\mathcal{L}_{\text{IV}}^6 = & \frac{\lambda_{iijk}^H}{\Lambda^2} \mathcal{O}_{iijk}^H + \frac{\lambda_{ijik}^H}{\Lambda^2} \mathcal{O}_{ijik}^H + \frac{\lambda_{ijk i}^H}{\Lambda^2} \mathcal{O}_{ijk i}^H \\
& + \frac{\lambda_{iijk}^r}{\Lambda^2} \mathcal{O}_{iijk}^r + \frac{\lambda_{jkii}^r}{\Lambda^2} \mathcal{O}_{jkii}^r + \frac{\lambda_{ijik}^r}{\Lambda^2} \mathcal{O}_{ijik}^r + \frac{\lambda_{ikij}^r}{\Lambda^2} \mathcal{O}_{ikij}^r + \frac{\lambda_{ijk i}^r}{\Lambda^2} \mathcal{O}_{ijk i}^r + \frac{\lambda_{k i i j}^r}{\Lambda^2} \mathcal{O}_{k i i j}^r \\
& + \frac{\lambda_{iijk}^T}{\Lambda^2} \mathcal{O}_{iijk}^T + \frac{\lambda_{ijik}^T}{\Lambda^2} \mathcal{O}_{ijik}^T + \frac{\lambda_{ijk i}^T}{\Lambda^2} \mathcal{O}_{ijk i}^T \\
& + \frac{\lambda_{iijk}^{HT}}{\Lambda^2} \mathcal{O}_{iijk}^{HT} + \frac{\lambda_{jkii}^{HT}}{\Lambda^2} \mathcal{O}_{jkii}^{HT} + \frac{\lambda_{ijik}^{HT}}{\Lambda^2} \mathcal{O}_{ijik}^{HT} + \frac{\lambda_{ikij}^{HT}}{\Lambda^2} \mathcal{O}_{ikij}^{HT} + \frac{\lambda_{ijk i}^{HT}}{\Lambda^2} \mathcal{O}_{ijk i}^{HT} + \frac{\lambda_{k i i j}^{HT}}{\Lambda^2} \mathcal{O}_{k i i j}^{HT} + \text{H.c.} ,
\end{aligned} \tag{3.36}$$

where  $j < k$ . Since all of the coefficients can be complex, the Lagrangian has  $9N(N-1)(N-2)$  DOF for real and imaginary coefficients, respectively. In the case that the Higgs sector is governed by the nonlinear dynamics, the derivative interactions can be written as follows:

$$\mathcal{L}_{\text{IV}}^{6\text{NL}} = \frac{1}{f^2} \mathcal{T}_{abcd}^{\text{IV}} h^a h^b (\partial_\mu h^c) (\partial^\mu h^d), \tag{3.37}$$

where

$$\begin{aligned}
\mathcal{T}_{abcd}^{\text{IV}} = & 2a_{iik}^L \left( T_{(i,i)}^{L\alpha} \right)_{ac} \left( T_{(j,k)}^{L\alpha} \right)_{bd} + 2a_{ijk}^L \left( T_{(i,j)}^{L\alpha} \right)_{ac} \left( T_{(i,k)}^{L\alpha} \right)_{bd} + 2a_{iik}^R \left( T_{(i,i)}^{R\beta} \right)_{ac} \left( T_{(j,k)}^{R\beta} \right)_{bd} \\
& + 2a_{ijk}^R \left( T_{(i,j)}^{R\beta} \right)_{ac} \left( T_{(i,k)}^{R\beta} \right)_{bd} + 2a_{ijk}^S \left( S_{(i,j)}^{\alpha 3} \right)_{ac} \left( S_{(i,k)}^{\alpha 3} \right)_{bd} + 2a_{iik}^{SS} \left( S_{(i,j)}^{\alpha\beta} \right)_{ac} \left( S_{(i,k)}^{\alpha\beta} \right)_{bd} \\
& + 2a_{iik}^{LS} \left( T_{(i,i)}^{L\alpha} \right)_{ac} \left( S_{(j,k)}^{\alpha 3} \right)_{bd} + 2a_{ijk}^{LS} \left( T_{(i,j)}^{L\alpha} \right)_{ac} \left( S_{(i,k)}^{\alpha 3} \right)_{bd} + 2a_{ikij}^{LS} \left( T_{(i,k)}^{L\alpha} \right)_{ac} \left( S_{(i,j)}^{\alpha 3} \right)_{bd} \\
& + 2a_{iik}^Y \left( T_{(i,i)}^{R3} \right)_{ac} \left( T_{(j,k)}^{R3} \right)_{bd} + 2a_{ijk}^Y \left( T_{(i,j)}^{R3} \right)_{ac} \left( T_{(i,k)}^{R3} \right)_{bd} + 2a_{iik}^U \left( U_{(i,j)} \right)_{ac} \left( U_{(i,k)} \right)_{bd} \\
& + 2a_{iik}^{YU} \left( T_{(i,i)}^{R3} \right)_{ac} \left( U_{(j,k)} \right)_{bd} + 2a_{ijk}^{YU} \left( T_{(i,j)}^{R3} \right)_{ac} \left( U_{(i,k)} \right)_{bd} + 2a_{ikij}^{YU} \left( T_{(i,k)}^{R3} \right)_{ac} \left( U_{(i,j)} \right)_{bd}.
\end{aligned} \tag{3.38}$$

Equation (3.37) can be expanded in terms of the  $SU(2)$  doublets:

$$\begin{aligned}
\mathcal{L}_{\text{IV}}^{\text{6NL}} = & \frac{c_{[ijk]}^{\text{IV1}}}{2f^2} O_{iik}^H + \frac{c_{[ijk]}^{\text{IV2}}}{2f^2} O_{ijk}^H + \frac{c_{[ijk]}^{\text{IV3}}}{2f^2} O_{ijk}^H \\
& - \frac{c_{[ijk]}^{\text{IV1}}}{f^2} (O_{iik}^r + O_{jki}^r) - \frac{c_{[ijk]}^{\text{IV2}}}{f^2} (O_{ijk}^r + O_{ikj}^r) - \frac{c_{[ijk]}^{\text{IV3}}}{f^2} (O_{ijk}^r + O_{kij}^r) \\
& + \frac{c_{[ijk]}^{\text{IV4}}}{2f^2} O_{iik}^T + \frac{c_{[ijk]}^{\text{IV5}}}{2f^2} O_{ijk}^T + \frac{c_{[ijk]}^{\text{IV6}}}{2f^2} O_{ijk}^T + \text{H.c.},
\end{aligned} \tag{3.39}$$

$$c_{[ijk]}^{\text{IV1}} = a_{iik}^L + a_{ijk}^R + a_{ijk}^S + 2a_{iik}^{SS} + i(-a_{iik}^{LS} + a_{ikij}^{LS}), \tag{3.40}$$

$$c_{[ijk]}^{\text{IV2}} = a_{iik}^L + a_{iik}^R - a_{ijk}^S - a_{iik}^{SS} - i(a_{ijk}^{LS} + a_{ikij}^{LS}), \tag{3.41}$$

$$c_{[ijk]}^{\text{IV3}} = a_{iik}^L + a_{ijk}^R - a_{iik}^{SS} + ia_{iik}^{LS}, \tag{3.42}$$

$$c_{[ijk]}^{\text{IV4}} = -a_{iik}^L + a_{iik}^L + a_{iik}^R - a_{ijk}^R + a_{ijk}^S + a_{iik}^Y + i(a_{iik}^{LS} - a_{ijk}^{LS} + a_{ikij}^{LS} - a_{iik}^{YU}), \tag{3.43}$$

$$c_{[ijk]}^{\text{IV5}} = -a_{iik}^R + a_{iik}^R + a_{ijk}^Y - a_{ijk}^U - i(a_{ijk}^{YU} + a_{ikij}^{YU}), \tag{3.44}$$

$$c_{[ijk]}^{\text{IV6}} = a_{iik}^L - a_{iik}^L - a_{ijk}^S + a_{ijk}^Y + a_{ijk}^U + i(a_{iik}^{LS} - a_{ijk}^{LS} + a_{ikij}^{LS} + a_{ijk}^{YU} - a_{ikij}^{YU}). \tag{3.45}$$

The last Lagrangian has  $3N(N-1)(N-2)$  real and  $3N(N-1)(N-2)$  imaginary DOF. Namely, each DOF is reduced to one third of the original one in the Lagrangian (3.36) (Table 4).

	Re	Im
General	$9N(N-1)(N-2)$	$9N(N-1)(N-2)$
$\mathcal{O}^H$	$(3/2)N(N-1)(N-2)$	$(3/2)N(N-1)(N-2)$
$\mathcal{O}^T$	$(3/2)N(N-1)(N-2)$	$(3/2)N(N-1)(N-2)$
$\mathcal{O}^r$	$3N(N-1)(N-2)$	$3N(N-1)(N-2)$
$\mathcal{O}^{HT}$	$3N(N-1)(N-2)$	$3N(N-1)(N-2)$
Nonlinear	$3N(N-1)(N-2)$	$3N(N-1)(N-2)$

**Table 4.** The number of the independent dimension-six derivative interactions for type IV. The first and second columns mean real and imaginary DOF, respectively. We show the number of DOF for the case where  $SU(2)_L \times U(1)_Y$  invariance is imposed (General) and the case with nonlinear realization (Nonlinear). In the case of General, the DOF of each type of operators is also shown.

### 3.5 Type V

Type V derivative interactions are described by the Lagrangian below:

$$\begin{aligned}
\mathcal{L}_V^6 = & \frac{\lambda_{ijkl}^H}{\Lambda^2} O_{ijkl}^H + \frac{\lambda_{ijlk}^H}{\Lambda^2} O_{ijlk}^H + \frac{\lambda_{ikjl}^H}{\Lambda^2} O_{ikjl}^H + \frac{\lambda_{iklj}^H}{\Lambda^2} O_{iklj}^H + \frac{\lambda_{iljk}^H}{\Lambda^2} O_{iljk}^H + \frac{\lambda_{ilkj}^H}{\Lambda^2} O_{ilkj}^H \\
& + \frac{\lambda_{ijkl}^r}{\Lambda^2} O_{ijkl}^r + \frac{\lambda_{ijlk}^r}{\Lambda^2} O_{ijlk}^r + \frac{\lambda_{ikjl}^r}{\Lambda^2} O_{ikjl}^r + \frac{\lambda_{iklj}^r}{\Lambda^2} O_{iklj}^r + \frac{\lambda_{iljk}^r}{\Lambda^2} O_{iljk}^r + \frac{\lambda_{ilkj}^r}{\Lambda^2} O_{ilkj}^r \\
& + \frac{\lambda_{jkil}^r}{\Lambda^2} O_{jkil}^r + \frac{\lambda_{jkli}^r}{\Lambda^2} O_{jkli}^r + \frac{\lambda_{jlik}^r}{\Lambda^2} O_{jlik}^r + \frac{\lambda_{jlkj}^r}{\Lambda^2} O_{jlkj}^r + \frac{\lambda_{klij}^r}{\Lambda^2} O_{klij}^r + \frac{\lambda_{klji}^r}{\Lambda^2} O_{klji}^r \\
& + \frac{\lambda_{ijkl}^T}{\Lambda^2} O_{ijkl}^T + \frac{\lambda_{ijik}^T}{\Lambda^2} O_{ijik}^T + \frac{\lambda_{ikjl}^T}{\Lambda^2} O_{ikjl}^T + \frac{\lambda_{iklj}^T}{\Lambda^2} O_{iklj}^T + \frac{\lambda_{iljk}^T}{\Lambda^2} O_{iljk}^T + \frac{\lambda_{ilkj}^T}{\Lambda^2} O_{ilkj}^T \\
& + \frac{\lambda_{ijkl}^{HT}}{\Lambda^2} O_{ijkl}^{HT} + \frac{\lambda_{ijlk}^{HT}}{\Lambda^2} O_{ijlk}^{HT} + \frac{\lambda_{ikjl}^{HT}}{\Lambda^2} O_{ikjl}^{HT} + \frac{\lambda_{iklj}^{HT}}{\Lambda^2} O_{iklj}^{HT} + \frac{\lambda_{iljk}^{HT}}{\Lambda^2} O_{iljk}^{HT} + \frac{\lambda_{ilkj}^{HT}}{\Lambda^2} O_{ilkj}^{HT} \\
& + \frac{\lambda_{jkil}^{HT}}{\Lambda^2} O_{jkil}^{HT} + \frac{\lambda_{jkli}^{HT}}{\Lambda^2} O_{jkli}^{HT} + \frac{\lambda_{jlik}^{HT}}{\Lambda^2} O_{jlik}^{HT} + \frac{\lambda_{jlkj}^{HT}}{\Lambda^2} O_{jlkj}^{HT} + \frac{\lambda_{klij}^{HT}}{\Lambda^2} O_{klij}^{HT} + \frac{\lambda_{klji}^{HT}}{\Lambda^2} O_{klji}^{HT} + \text{H.c.} ,
\end{aligned} \tag{3.46}$$

where  $i < j < k < l$ . The Lagrangian has  $3N(N-1)(N-2)(N-3)$  DOF, and real (imaginary) DOF is a half of them. Similarly to previous sections, let us study the derivative interactions with the constraint of the nonlinear realization:

$$\mathcal{L}_V^{6\text{NL}} = \frac{1}{f^2} \mathcal{T}_{abcd}^V h^a h^b (\partial_\mu h^c) (\partial^\mu h^d), \tag{3.47}$$

where

$$\begin{aligned}
\mathcal{T}_{abcd}^V = & 2a_{ijkl}^L \left( T_{(i,j)}^{L\alpha} \right)_{ac} \left( T_{(k,l)}^{L\alpha} \right)_{bd} + 2a_{ikjl}^L \left( T_{(i,k)}^{L\alpha} \right)_{ac} \left( T_{(j,l)}^{L\alpha} \right)_{bd} + 2a_{iljk}^L \left( T_{(i,l)}^{L\alpha} \right)_{ac} \left( T_{(j,k)}^{L\alpha} \right)_{bd} \\
& + 2a_{ijkl}^R \left( T_{(i,j)}^{R\beta} \right)_{ac} \left( T_{(k,l)}^{R\beta} \right)_{bd} + 2a_{ikjl}^R \left( T_{(i,k)}^{R\beta} \right)_{ac} \left( T_{(j,l)}^{R\beta} \right)_{bd} + 2a_{iljk}^R \left( T_{(i,l)}^{R\beta} \right)_{ac} \left( T_{(j,k)}^{R\beta} \right)_{bd} \\
& + 2a_{ijkl}^S \left( S_{(i,j)}^{\alpha 3} \right)_{ac} \left( S_{(k,l)}^{\alpha 3} \right)_{bd} + 2a_{ikjl}^S \left( S_{(i,k)}^{\alpha 3} \right)_{ac} \left( S_{(j,l)}^{\alpha 3} \right)_{bd} + 2a_{iljk}^S \left( S_{(i,l)}^{\alpha 3} \right)_{ac} \left( S_{(j,k)}^{\alpha 3} \right)_{bd} \\
& + 2a_{ijkl}^{SS} \left( S_{(i,j)}^{\alpha\beta} \right)_{ac} \left( S_{(k,l)}^{\alpha\beta} \right)_{bd} + 2a_{ikjl}^{SS} \left( S_{(i,k)}^{\alpha\beta} \right)_{ac} \left( S_{(j,l)}^{\alpha\beta} \right)_{bd} + 2a_{iljk}^{SS} \left( S_{(i,l)}^{\alpha\beta} \right)_{ac} \left( S_{(j,k)}^{\alpha\beta} \right)_{bd} \\
& + 2a_{ijkl}^{LS} \left( T_{(i,j)}^{L\alpha} \right)_{ac} \left( S_{(k,l)}^{\alpha 3} \right)_{bd} + 2a_{ikjl}^{LS} \left( T_{(i,k)}^{L\alpha} \right)_{ac} \left( S_{(j,l)}^{\alpha 3} \right)_{bd} + 2a_{iljk}^{LS} \left( T_{(i,l)}^{L\alpha} \right)_{ac} \left( S_{(j,k)}^{\alpha 3} \right)_{bd} \\
& + 2a_{klij}^{LS} \left( T_{(k,l)}^{L\alpha} \right)_{ac} \left( S_{(i,j)}^{\alpha 3} \right)_{bd} + 2a_{jlik}^{LS} \left( T_{(j,l)}^{L\alpha} \right)_{ac} \left( S_{(i,k)}^{\alpha 3} \right)_{bd} + 2a_{jkil}^{LS} \left( T_{(j,k)}^{L\alpha} \right)_{ac} \left( S_{(i,l)}^{\alpha 3} \right)_{bd} \\
& + 2a_{ijkl}^Y \left( T_{(i,j)}^{R3} \right)_{ac} \left( T_{(k,l)}^{R3} \right)_{bd} + 2a_{ikjl}^Y \left( T_{(i,k)}^{R3} \right)_{ac} \left( T_{(j,l)}^{R3} \right)_{bd} + 2a_{iljk}^Y \left( T_{(i,l)}^{R3} \right)_{ac} \left( T_{(j,k)}^{R3} \right)_{bd} \\
& + 2a_{ijkl}^U \left( U_{(i,j)} \right)_{ac} \left( U_{(k,l)} \right)_{bd} + 2a_{ikjl}^U \left( U_{(i,k)} \right)_{ac} \left( U_{(j,l)} \right)_{bd} + 2a_{iljk}^U \left( U_{(i,l)} \right)_{ac} \left( U_{(j,k)} \right)_{bd} \\
& + 2a_{ijkl}^{YU} \left( T_{(i,j)}^{R3} \right)_{ac} \left( U_{(k,l)} \right)_{bd} + 2a_{ikjl}^{YU} \left( T_{(i,k)}^{R3} \right)_{ac} \left( U_{(j,l)} \right)_{bd} + 2a_{iljk}^{YU} \left( T_{(i,l)}^{R3} \right)_{ac} \left( U_{(j,k)} \right)_{bd} \\
& + 2a_{klij}^{YU} \left( T_{(k,l)}^{R3} \right)_{ac} \left( U_{(i,j)} \right)_{bd} + 2a_{jlik}^{YU} \left( T_{(j,l)}^{R3} \right)_{ac} \left( U_{(i,k)} \right)_{bd} + 2a_{jkil}^{YU} \left( T_{(j,k)}^{R3} \right)_{ac} \left( U_{(i,l)} \right)_{bd} .
\end{aligned} \tag{3.48}$$

Finally, we obtain the derivative interactions of the Higgs bosons realized as the NG bosons below:

$$\begin{aligned}
\mathcal{L}_V^{6\text{NL}} = & \frac{c_{[ijkl]}^{V1}}{2f^2} O_{ijkl}^H + \frac{c_{[ijkl]}^{V2}}{2f^2} O_{ijlk}^H + \frac{c_{[ijkl]}^{V3}}{2f^2} O_{ikjl}^H + \frac{c_{[ijkl]}^{V4}}{2f^2} O_{iklj}^H + \frac{c_{[ijkl]}^{V5}}{2f^2} O_{iljk}^H + \frac{c_{[ijkl]}^{V6}}{2f^2} O_{ilkj}^H \\
& - \frac{c_{[ijkl]}^{V1}}{f^2} (O_{ijkl}^r + O_{klji}^r) - \frac{c_{[ijkl]}^{V2}}{f^2} (O_{ijlk}^r + O_{klji}^r) - \frac{c_{[ijkl]}^{V3}}{f^2} (O_{ikjl}^r + O_{jlki}^r) \\
& - \frac{c_{[ijkl]}^{V4}}{f^2} (O_{iklj}^r + O_{jlki}^r) - \frac{c_{[ijkl]}^{V5}}{f^2} (O_{iljk}^r + O_{jkil}^r) - \frac{c_{[ijkl]}^{V6}}{f^2} (O_{ilkj}^r + O_{jkli}^r) \\
& + \frac{c_{[ijkl]}^{V7}}{2f^2} O_{ijkl}^T + \frac{c_{[ijkl]}^{V8}}{2f^2} O_{ijlk}^T + \frac{c_{[ijkl]}^{V9}}{2f^2} O_{ikjl}^T + \frac{c_{[ijkl]}^{V10}}{2f^2} O_{iklj}^T + \frac{c_{[ijkl]}^{V11}}{2f^2} O_{iljk}^T + \frac{c_{[ijkl]}^{V12}}{2f^2} O_{ilkj}^T + \text{H.c.} ,
\end{aligned} \tag{3.49}$$

$$c_{[ijkl]}^{V1} = a_{iljk}^L + a_{ikjl}^R + a_{iljk}^S + a_{ikjl}^{SS} + a_{iljk}^{SS} + i(a_{iljk}^{LS} - a_{jkil}^{LS}), \quad (3.50)$$

$$c_{[ijkl]}^{V2} = a_{ikjl}^L + a_{iljk}^R + a_{ikjl}^S + a_{ikjl}^{SS} + a_{iljk}^{SS} + i(a_{ikjl}^{LS} - a_{jlik}^{LS}), \quad (3.51)$$

$$c_{[ijkl]}^{V3} = a_{iljk}^L + a_{ijk}^R - a_{iljk}^S + a_{ijk}^{SS} - a_{iljk}^{SS} - i(a_{iljk}^{LS} + a_{jkil}^{LS}), \quad (3.52)$$

$$c_{[ijkl]}^{V4} = a_{iljk}^L + a_{iljk}^R + a_{iljk}^S + a_{ijk}^{SS} - a_{iljk}^{SS} + i(a_{iljk}^{LS} - a_{klj}^{LS}), \quad (3.53)$$

$$c_{[ijkl]}^{V5} = a_{ikjl}^L + a_{ijk}^R - a_{ikjl}^S - a_{ijk}^{SS} - a_{ikjl}^{SS} - i(a_{ikjl}^{LS} + a_{jlik}^{LS}), \quad (3.54)$$

$$c_{[ijkl]}^{V6} = a_{iljk}^L + a_{ikjl}^R - a_{iljk}^S - a_{ijk}^{SS} - a_{ikjl}^{SS} - i(a_{iljk}^{LS} + a_{klj}^{LS}), \quad (3.55)$$

$$c_{[ijkl]}^{V7} = -a_{iljk}^L + a_{iljk}^L + a_{ijk}^R - a_{ikjl}^R + a_{ijk}^Y - a_{iljk}^U + a_{ijk}^S + a_{iljk}^S + a_{ijk}^{SS} - a_{ikjl}^{SS} + a_{iljk}^{SS} \\ + i(-a_{ijk}^{YU} - a_{klj}^{YU} + a_{ijk}^{LS} + a_{iljk}^{LS} + a_{klj}^{LS} - a_{jkil}^{LS}), \quad (3.56)$$

$$c_{[ijkl]}^{V8} = -a_{iljk}^L + a_{ikjl}^L + a_{ijk}^R - a_{iljk}^R + a_{ijk}^Y + a_{iljk}^U - a_{ijk}^S + a_{ikjl}^S - a_{ijk}^{SS} + a_{ikjl}^{SS} - a_{iljk}^{SS} \\ + i(a_{ijk}^{YU} - a_{klj}^{YU} - a_{ijk}^{LS} - a_{ikjl}^{LS} + a_{klj}^{LS} - a_{jlik}^{LS}), \quad (3.57)$$

$$c_{[ijkl]}^{V9} = -a_{ikjl}^L + a_{iljk}^L - a_{ijk}^R + a_{ikjl}^R + a_{ijk}^Y - a_{ikjl}^U + a_{ijk}^S - a_{iljk}^S - a_{ijk}^{SS} + a_{ikjl}^{SS} - a_{iljk}^{SS} \\ + i(-a_{ikjl}^{YU} - a_{jlik}^{YU} + a_{ikjl}^{LS} - a_{iljk}^{LS} + a_{jlik}^{LS} - a_{jkil}^{LS}), \quad (3.58)$$

$$c_{[ijkl]}^{V10} = a_{iljk}^L - a_{ikjl}^L + a_{ikjl}^R - a_{iljk}^R + a_{ijk}^Y + a_{iljk}^U - a_{ijk}^S - a_{ikjl}^S + a_{ijk}^{SS} - a_{ikjl}^{SS} + a_{iljk}^{SS} \\ + i(a_{ikjl}^{YU} - a_{jlik}^{YU} + a_{ijk}^{LS} - a_{ikjl}^{LS} - a_{klj}^{LS} + a_{jlik}^{LS}), \quad (3.59)$$

$$c_{[ijkl]}^{V11} = a_{ikjl}^L - a_{iljk}^L - a_{ijk}^R + a_{iljk}^R + a_{iljk}^Y - a_{iljk}^U - a_{ikjl}^S + a_{iljk}^S + a_{ijk}^{SS} - a_{ikjl}^{SS} + a_{iljk}^{SS} \\ + i(-a_{iljk}^{YU} - a_{jkil}^{YU} - a_{ikjl}^{LS} + a_{iljk}^{LS} - a_{jlik}^{LS} + a_{jkil}^{LS}), \quad (3.60)$$

$$c_{[ijkl]}^{V12} = a_{ijk}^L - a_{iljk}^L - a_{ikjl}^R + a_{iljk}^R + a_{iljk}^Y + a_{iljk}^U - a_{ijk}^S - a_{iljk}^S - a_{ijk}^{SS} + a_{ikjl}^{SS} - a_{iljk}^{SS} \\ + i(a_{iljk}^{YU} - a_{jkil}^{YU} - a_{ijk}^{LS} - a_{iljk}^{LS} - a_{klj}^{LS} + a_{jlik}^{LS}). \quad (3.61)$$

There are  $(1/2)N(N-1)(N-2)(N-3)$  real DOF, and the imaginary DOF is the same as the real one. The DOF becomes one third of original one in either case (Table 5).

	Re	Im
General	$(3/2)N(N-1)(N-2)(N-3)$	$(3/2)N(N-1)(N-2)(N-3)$
$\mathcal{O}^H$	$(1/4)N(N-1)(N-2)(N-3)$	$(1/4)N(N-1)(N-2)(N-3)$
$\mathcal{O}^T$	$(1/4)N(N-1)(N-2)(N-3)$	$(1/4)N(N-1)(N-2)(N-3)$
$\mathcal{O}^r$	$(1/2)N(N-1)(N-2)(N-3)$	$(1/2)N(N-1)(N-2)(N-3)$
$\mathcal{O}^{HT}$	$(1/2)N(N-1)(N-2)(N-3)$	$(1/2)N(N-1)(N-2)(N-3)$
Nonlinear	$(1/2)N(N-1)(N-2)(N-3)$	$(1/2)N(N-1)(N-2)(N-3)$

**Table 5.** The number of the independent dimension-six derivative interactions for type V. The first and second columns mean real and imaginary DOF, respectively. We show the number of DOF for the case where  $SU(2)_L \times U(1)_Y$  invariance is imposed (General) and the case with nonlinear realization (Nonlinear). In the case of General, the DOF of each type of operators is also shown.

As a conclusion of this section, we discuss the number of independent derivative interactions in the case of the NHDM. Summing up the DOF given by this section, there are  $(1/2)N^2(N^2+3)$  real and  $(1/2)N^2(N^2-1)$  imaginary DOF for models where Higgs



doublets are generated by nonlinear dynamics, while general NHDM have  $(3/2)N^2(N^2 + 1)$  real and  $(1/2)N^2(3N^2 - 1)$  imaginary coefficients (Table 6). If  $N$  is large enough, the DOF is about one third compared with the original one.

	Re	Im
General	$(3/2)N^2(N^2 + 1)$	$(1/2)N^2(3N^2 - 1)$
Nonlinear	$(1/2)N^2(N^2 + 3)$	$(1/2)N^2(N^2 - 1)$

**Table 6.** The number of the independent dimension-six derivative operators for NHDMs. The first and second columns mean real and imaginary DOF, respectively. The first and second rows correspond to the models imposing only  $SU(2)_L \times U(1)_Y$  and nonlinear dynamics, respectively.

## 4 Application to two Higgs doublet models

In the previous section, we have derived the general expression of the dimension-six derivative interactions for the NHDM, assuming the Higgs bosons as PGBs. Its phenomenological consequences are studied for the 2HDM in this section as an explicit example. Many models which include the two Higgs doublets as NG fields are proposed [12, 13]. In particular,  $SO(4)$  invariant dimension-six derivative interactions are examined in Ref. [13].

We discuss the scalar four point interactions with two derivatives,  $\mathcal{O}^H, \mathcal{O}^T$  and  $\mathcal{O}^r$ . The operator,  $\mathcal{O}^{HT}$ , does not appear in the case of the nonlinear dynamics. Since the Higgs doublets include the longitudinal modes of the gauge bosons and the physical Higgs bosons, the interactions contribute to the vector boson fusion processes in high energy region. As we show in the following, the rising behavior of the amplitudes is determined by these derivative operators in high energy region if we neglect  $\mathcal{O}(v^2/f^2)$  corrections from the EWSB.

Finally, cross sections of vector boson fusion processes are presented in the case where the custodial symmetry is imposed. We show that a relation between cross sections of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  and  $W_L^+ W_L^- \rightarrow hh$  in the SILH is violated by additional parameters.

#### 4.1 Dimension-six derivative interactions

The derivative interactions on the 2HDM governed by nonlinear dynamics can be written using interactions of type I, II and III:

$$\begin{aligned}
\mathcal{T}_{2\text{HDM}}^{abcd} = & a_{1111}^L \left( T_{(1,1)}^{L\alpha} \right)_{ac} \left( T_{(1,1)}^{L\alpha} \right)_{bd} + 2a_{1112}^L \left( T_{(1,1)}^{L\alpha} \right)_{ac} \left( T_{(1,2)}^{L\alpha} \right)_{bd} + 2a_{1122}^L \left( T_{(1,1)}^{L\alpha} \right)_{ac} \left( T_{(2,2)}^{L\alpha} \right)_{bd} \\
& + a_{1212}^L \left( T_{(1,2)}^{L\alpha} \right)_{ac} \left( T_{(1,2)}^{L\alpha} \right)_{bd} + 2a_{2221}^L \left( T_{(2,2)}^{L\alpha} \right)_{ac} \left( T_{(2,1)}^{L\alpha} \right)_{bd} + a_{2222}^L \left( T_{(2,2)}^{L\alpha} \right)_{ac} \left( T_{(2,2)}^{L\alpha} \right)_{bd} \\
& + a_{1111}^R \left( T_{(1,1)}^{R\alpha} \right)_{ac} \left( T_{(1,1)}^{R\alpha} \right)_{bd} + 2a_{1112}^R \left( T_{(1,1)}^{R\alpha} \right)_{ac} \left( T_{(1,2)}^{R\alpha} \right)_{bd} + 2a_{1122}^R \left( T_{(1,1)}^{R\alpha} \right)_{ac} \left( T_{(2,2)}^{R\alpha} \right)_{bd} \\
& + a_{1212}^R \left( T_{(1,2)}^{R\alpha} \right)_{ac} \left( T_{(1,2)}^{R\alpha} \right)_{bd} + 2a_{2221}^R \left( T_{(2,2)}^{R\alpha} \right)_{ac} \left( T_{(2,1)}^{R\alpha} \right)_{bd} + a_{2222}^R \left( T_{(2,2)}^{R\alpha} \right)_{ac} \left( T_{(2,2)}^{R\alpha} \right)_{bd} \\
& + a_{1212}^S \left( S_{(1,2)}^{\alpha 3} \right)_{ac} \left( S_{(1,2)}^{\alpha 3} \right)_{bd} + a_{1212}^{SS} \left( S_{(1,2)}^{\alpha\beta} \right)_{ac} \left( S_{(1,2)}^{\alpha\beta} \right)_{bd} + 2a_{1112}^{LS} \left( T_{(1,1)}^{L\alpha} \right)_{ac} \left( S_{(1,2)}^{\alpha 3} \right)_{bd} \\
& + a_{1212}^{LS} \left( T_{(1,2)}^{L\alpha} \right)_{ac} \left( S_{(1,2)}^{\alpha 3} \right)_{bd} + 2a_{2221}^{LS} \left( T_{(2,2)}^{L\alpha} \right)_{ac} \left( S_{(2,1)}^{\alpha 3} \right)_{bd} + 2a_{1111}^Y \left( T_{(1,1)}^{R3} \right)_{ac} \left( T_{(1,1)}^{R3} \right)_{bd} \\
& + 4a_{1112}^Y \left( T_{(1,1)}^{R3} \right)_{ac} \left( T_{(1,2)}^{R3} \right)_{bd} + 2a_{1122}^Y \left( T_{(1,1)}^{R3} \right)_{ac} \left( T_{(2,2)}^{R3} \right)_{bd} + a_{1212}^Y \left( T_{(1,2)}^{R3} \right)_{ac} \left( T_{(1,2)}^{R3} \right)_{bd} \\
& + 4a_{2212}^Y \left( T_{(2,2)}^{R3} \right)_{ac} \left( T_{(1,2)}^{R3} \right)_{bd} + 2a_{2222}^Y \left( T_{(2,2)}^{R3} \right)_{ac} \left( T_{(2,2)}^{R3} \right)_{bd} + a_{1212}^U \left( U_{(1,2)} \right)_{ac} \left( U_{(1,2)} \right)_{bd} \\
& + 4a_{1112}^{YU} \left( T_{(1,1)}^{R3} \right)_{ac} \left( U_{(1,2)} \right)_{bd} + a_{1212}^{YU} \left( T_{(1,2)}^{R3} \right)_{ac} \left( U_{(1,2)} \right)_{bd} + 4a_{2212}^{YU} \left( T_{(2,2)}^{R3} \right)_{ac} \left( U_{(1,2)} \right)_{bd} .
\end{aligned} \tag{4.1}$$

In the following, we assume that the Lagrangian possesses the custodial invariance which automatically ensures the  $CP$  invariance [14]<sup>2</sup>. From the phenomenological standpoint, the custodial symmetry breaking terms are severely constrained. For the case with custodial symmetry violating terms, see Appendix C. The  $SO(4)$  invariance is achieved by the following conditions:

$$a_{1112}^{YU} = 0, \quad a_{1212}^{YU} = 0, \quad a_{2212}^{YU} = 0, \tag{4.2}$$

$$a_{1112}^{LS} = 0, \quad a_{1212}^{LS} = 0, \quad a_{2212}^{LS} = 0, \tag{4.3}$$

$$a_{1111}^Y = 0, \quad a_{1112}^Y = 0, \quad a_{2212}^Y = 0, \quad a_{2222}^Y = 0, \tag{4.4}$$

$$a_{1212}^S = a_{1212}^Y = -\frac{a_{1122}^Y}{2}. \tag{4.5}$$

Note that there exists a nontrivial realization of the  $SO(4)$  invariance even if each term corresponding to the coefficient appearing in Eq. (4.5) violates the  $SO(4)$  symmetry.

In terms of the  $SU(2)_L$  doublet, since  $\mathcal{O}^r$  can be eliminated by the field redefinition

---

<sup>2</sup> In Ref. [14], it is pointed out the custodial symmetry of the Higgs potential is violated by the imaginary part of  $H_i^\dagger H_j$ . Since the symmetry in the derivative interactions is violated by the imaginary part of  $H_i^\dagger \partial H_j$ , their discussion can be applied to this case by the replacement of  $\partial H_k$  with  $H_l$  ( $k \neq l$ ).

(see Appendix D), we obtain the following Lagrangian:

$$\begin{aligned}\mathcal{L}_{2\text{HDM}}^6 = & \frac{c_{1111}^H}{2f^2}O_{1111}^H + \frac{c_{1112}^H}{f^2}(O_{1112}^H + O_{1121}^H) + \frac{c_{1122}^H}{f^2}O_{1122}^H + \frac{c_{1221}^H}{f^2}O_{1221}^H \\ & + \frac{c_{1212}^H}{2f^2}(O_{1212}^H + O_{2121}^H) + \frac{c_{2221}^H}{f^2}(O_{2212}^H + O_{2221}^H) + \frac{c_{2222}^H}{2f^2}O_{2222}^H \\ & + \frac{c_{1122}^T}{f^2}O_{1122}^T + \frac{c_{1221}^T}{f^2}O_{1221}^T + \frac{c_{1212}^T}{2f^2}(O_{1212}^T + O_{2121}^T).\end{aligned}\quad (4.6)$$

The relations between the above coefficients and the ones in Eq. (4.1) are

$$c_{1111}^H = \frac{3}{2}(a_{1111}^L + a_{1111}^R), \quad (4.7)$$

$$c_{1112}^H = \frac{3}{2}(a_{1112}^L + a_{1112}^R), \quad (4.8)$$

$$c_{1122}^H = \frac{3}{2}(a_{1212}^L + a_{1212}^R + a_{1212}^S + 2a_{1212}^{SS}), \quad (4.9)$$

$$c_{1221}^H = \frac{3}{2}(a_{1122}^L + a_{1212}^R - a_{1212}^{SS}), \quad (4.10)$$

$$c_{1212}^H = \frac{3}{2}(a_{1212}^L + a_{1122}^R - a_{1212}^S - a_{1212}^{SS}), \quad (4.11)$$

$$c_{2221}^H = \frac{3}{2}(a_{2221}^L + a_{2221}^R), \quad (4.12)$$

$$c_{2222}^H = \frac{3}{2}(a_{2222}^L + a_{2222}^R), \quad (4.13)$$

$$c_{1122}^T = \frac{1}{2}(-a_{1122}^L + a_{1212}^L + a_{1122}^R - a_{1212}^R - a_{1212}^S), \quad (4.14)$$

$$c_{1221}^T = \frac{1}{2}(-a_{1212}^L + a_{1122}^L + a_{1212}^U), \quad (4.15)$$

$$c_{1212}^T = \frac{1}{2}(a_{1212}^R - a_{1122}^R + a_{1212}^S - a_{1212}^U). \quad (4.16)$$

The other coefficients vanish. In the above equations, the following relations are obtained:

$$c_{1122}^T = -(c_{1221}^T + c_{1212}^T) = -\frac{1}{3}(c_{1221}^H - c_{1212}^H). \quad (4.17)$$

Therefore, the Lagrangian discussed here has eight real DOF. This is equivalent to a result in Ref. [13]. On the other hand, using equations in Table 6, the general 2HDM realized by nonlinear dynamics without the  $SO(4)$  symmetry has 14 real and six imaginary DOF. For the following discussions, we eliminate  $c_{1122}^T$  and  $c_{1212}^T$  using Eq. (4.17).

We first discuss the kinetic term mixing induced by the dimension-six derivative interactions and show that this mixing can be neglected in deriving leading behavior of the scattering amplitudes at high energy.

After the EWSB, the kinetic term mixing appears in the Lagrangian for neutral Higgs bosons at  $\mathcal{O}(v^2/f^2)$ . Since we assume the  $CP$  invariance, the mixing appears for the  $CP$  even and odd sector separately. We show the prescription for the mixing with the  $CP$  even sector as an example. The kinetic term and the mass term of the  $CP$  even sector after the

EWSB can be written as

$$\mathcal{L}_K = \frac{1}{2} \begin{pmatrix} \partial^\mu h_0^3 & \partial^\mu h_0^7 \end{pmatrix} (\mathbf{1}_2 + C_K) \begin{pmatrix} \partial_\mu h_0^3 \\ \partial_\mu h_0^7 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} h_0^3 & h_0^7 \end{pmatrix} M \begin{pmatrix} h_0^3 \\ h_0^7 \end{pmatrix}, \quad (4.18)$$

where  $h^3 = v \cos \beta + h_0^3$ ,  $h^7 = v \sin \beta + h_0^7$  and the matrix  $C_K$  stands for the  $\mathcal{O}(v^2/f^2)$  corrections. The mass matrix,  $M$ , includes the effects from dimension-six Higgs potentials. The following matrices are introduced to make the kinetic term to be the canonical form:

$$V = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad K = \begin{pmatrix} 1/\sqrt{1+k_1} & 0 \\ 0 & 1/\sqrt{1+k_2} \end{pmatrix}, \quad (4.19)$$

such that,

$$VC_K V^\dagger = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}. \quad (4.20)$$

Mass eigenstates of the  $CP$  even Higgs bosons,  $h$  and  $H$  ( $m_h \leq m_H$ ), can be written as

$$\begin{pmatrix} h \\ H \end{pmatrix} = UV^\dagger K^{-1} V \begin{pmatrix} h_0^3 \\ h_0^7 \end{pmatrix}, \quad (4.21)$$

where  $U$  diagonalizes the mass matrix,  $V^\dagger K V M V^\dagger K V$ , in the basis giving the canonical kinetic term. Using the mixing angle  $\alpha$  which diagonalizes the mass matrix  $M$  in Eq. (4.18), we can write

$$U = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} + U_\xi, \quad (4.22)$$

$$V^\dagger K V = \mathbf{1}_2 + K_\xi, \quad (4.23)$$

where  $U_\xi$  and  $K_\xi$  are  $\mathcal{O}(v^2/f^2)$  corrections given by the kinetic mixing. When we neglect  $\mathcal{O}(v^2/f^2)$  corrections, the mass eigenstates for the  $CP$  even sector are given as follows<sup>3</sup>:

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_0^3 \\ h_0^7 \end{pmatrix}. \quad (4.24)$$

As we mentioned, the dimension-six derivative interactions, Eq. (4.6), give the leading behavior of  $E^2/f^2$ , and the kinetic mixing only induces the correction of  $\mathcal{O}(v^2/f^2)$ .

Mass eigenstates of the  $CP$  odd sector,  $G^0$  and  $A$ , can be obtained similarly to the  $CP$  even sector by replacing  $\alpha$  with  $\beta$  for the diagonalization matrix at the leading order:

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} h^4 \\ h^8 \end{pmatrix}, \quad (4.25)$$

where  $G^0$  is the longitudinal mode of  $Z$  boson and  $A$  is the  $CP$  odd Higgs boson. The matrix also gives us the mass eigenstates in the charged Higgs sector even if we take the  $\mathcal{O}(v^2/f^2)$  corrections into consideration because the dimension-six derivative interactions never contribute to the kinetic term of the charged Higgs sector after the field redefinition.

---

<sup>3</sup>This is a different notation from Ref. [15] in which  $CP$ -even mass eigenstates are defined as  $H = \cos \alpha h_0^3 + \sin \alpha h_0^7$  and  $h = -\sin \alpha h_0^3 + \cos \alpha h_0^7$ .

## 4.2 Scattering amplitudes of the longitudinal modes and the Higgs bosons

We derive the scattering amplitudes of the longitudinal modes and the Higgs bosons from dimension-six two-derivative interactions in the 2HDM governed by nonlinear dynamics with the custodial symmetry. Other contributions are neglected since they are subleading corrections in high energy region. To study the influence on vector boson fusion processes, we consider the case that the initial states consist of longitudinal modes of the SM gauge bosons,  $W_L^\pm$  and  $Z_L$ .

The amplitudes shown below are given in terms of the Mandelstam variables. For a process,  $V_1 V_2 \rightarrow X_1 X_2$ , the variables are defined as,

$$s = (p_1 + p_2)^2 = (q_1 + q_2)^2, \quad (4.26)$$

$$t = (q_1 - p_1)^2 = (q_2 - p_2)^2, \quad (4.27)$$

$$u = (q_2 - p_1)^2 = (q_1 - p_2)^2, \quad (4.28)$$

where  $p_1^\mu$ ,  $p_2^\mu$ ,  $q_1^\mu$  and  $q_2^\mu$  are momenta of the particles  $V_1$ ,  $V_2$ ,  $X_1$  and  $X_2$ , respectively. Since amplitudes are considered in the energy region which is much higher than the mass scale of the physical Higgs bosons, we show the results in terms of  $s$  and  $t$  using a relation,  $s + t + u = 0$ . The results of this subsection are based on the custodial invariance. In Appendix C, we also calculate scattering amplitudes including terms violating the custodial symmetry.

Firstly, amplitudes producing the SM particles,  $W_L^\pm$ ,  $Z_L$  and  $h$ , are presented. These processes also appear in the SILH model. In the following amplitudes, coefficients are given in Eq. (4.6). For notational simplicity, we define  $c_x = \cos x$ ,  $s_x = \sin x$ . The amplitudes are given by

$$\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}} = \frac{s+t}{f^2} C_1(\beta), \quad (4.29)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow hh)_{\text{cust}} = \frac{s}{f^2} C_2(\alpha, \beta), \quad (4.30)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L Z_L)_{\text{cust}} = \frac{s}{f^2} C_1(\beta), \quad (4.31)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow h Z_L)_{\text{cust}} = 0, \quad (4.32)$$

$$\mathcal{M}(Z_L Z_L \rightarrow W_L^+ W_L^-)_{\text{cust}} = \frac{s}{f^2} C_1(\beta), \quad (4.33)$$

$$\mathcal{M}(Z_L Z_L \rightarrow hh)_{\text{cust}} = \frac{s}{f^2} C_2(\alpha, \beta), \quad (4.34)$$

$$\mathcal{M}(Z_L Z_L \rightarrow Z_L Z_L)_{\text{cust}} = 0, \quad (4.35)$$

$$\mathcal{M}(Z_L Z_L \rightarrow h Z_L)_{\text{cust}} = 0, \quad (4.36)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ h)_{\text{cust}} = 0, \quad (4.37)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ Z_L)_{\text{cust}} = \frac{t}{f^2} C_1(\beta), \quad (4.38)$$

$$\mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ W_L^+)_{\text{cust}} = -\frac{s}{f^2} C_1(\beta), \quad (4.39)$$

where

$$C_1(\beta) = \frac{1}{8} \left( (3 + 4c_{2\beta} + c_{4\beta})c_{1111}^H + 4(2s_{2\beta} + s_{4\beta})c_{1112}^H \right. \\ \left. + 2(1 - c_{4\beta})(c_{1122}^H + c_{1221}^H + c_{1212}^H) \right. \\ \left. + 4(2s_{2\beta} - s_{4\beta})c_{2221}^H + (3 - 4c_{2\beta} + c_{4\beta})c_{2222}^H \right), \quad (4.40)$$

$$C_2(\alpha, \beta) = \frac{1}{8} \left( (2(1 + c_{2\beta}) + (1 + 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (2s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})c_{1111}^H \right. \\ \left. + 4(s_{2\beta} + (s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} + (c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})c_{1112}^H \right. \\ \left. + 2(2 - (1 + c_{4\beta})c_{2(\alpha-\beta)} + s_{4\beta}s_{2(\alpha-\beta)})c_{1122}^H \right. \\ \left. + 2((1 - c_{4\beta})c_{2(\alpha-\beta)} + s_{4\beta}s_{2(\alpha-\beta)})(c_{1221}^H + c_{1212}^H) \right. \\ \left. + 4(s_{2\beta} + (s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} + (c_{2\beta} - c_{4\beta})s_{2(\alpha-\beta)})c_{2221}^H \right. \\ \left. + (2(1 - c_{2\beta}) + (1 - 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} + (2s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})c_{2222}^H \right). \quad (4.41)$$

In the above expressions, we can see that five amplitudes, Eqs. (4.29), (4.31), (4.33), (4.38) and (4.39), are determined by  $C_1(\beta)$ , and two amplitudes, Eqs. (4.30) and (4.34), are determined by  $C_2(\alpha, \beta)$ . Among them, amplitudes in Eqs. (4.29) and (4.39) (Eqs. (4.31), (4.33) and (4.38)) are related by the crossing symmetry. If the custodial symmetry is broken, the amplitudes of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ ,  $W_L^+ W_L^- \rightarrow W_L^+ W_L^+$  and  $Z_L Z_L \rightarrow hh$  have  $c^T$  dependence as shown in Appendix C. Amplitudes in Eqs. (4.32) and (4.37) are also connected through the crossing symmetry. In particular, these amplitudes vanish when we require the custodial invariance.

We expand the coefficient  $C_2(\alpha, \beta)$  with  $\alpha - \beta$ :

$$C_2(\alpha, \beta) = C_1(\beta) \\ + 2(\alpha - \beta) \left( -(2s_{2\beta} + s_{4\beta})c_{1111}^H + 4(c_{2\beta} + c_{4\beta})c_{1112}^H \right. \\ \left. + 2s_{4\beta}(c_{1122}^H + c_{1221}^H + c_{1212}^H) + 4(c_{2\beta} - c_{4\beta})c_{2221}^H + (2s_{2\beta} - s_{4\beta})c_{2222}^H \right) \\ + \mathcal{O}((\alpha - \beta)^2) \quad (4.42)$$

In the SILH model, there is a simple relation among the amplitudes,  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ ,  $W_L^+ W_L^- \rightarrow Z_L Z_L$  and  $W_L^+ W_L^- \rightarrow hh$ , as shown in Eqs. (2.27) and (2.28). Such a relation is violated in the 2HDM because the process,  $W_L^+ W_L^- \rightarrow hh$ , depends on the parameter  $\alpha$  as well. However, if the decoupling limit,  $\alpha - \beta = 0^4$ , is imposed, the relation is recovered. In other words, even if we observe the scatterings including only the SM particles in the high energy region, the SILH model can be discriminated from the models involving several Higgs doublets.

Secondly, we show amplitudes including one heavy Higgs boson,  $H^\pm$ ,  $H$  or  $A$ , in the final states. If they are heavier than the SM ones, then the following behavior can be derived in the energy region much higher than the mass scale of heavy Higgs bosons. The amplitudes are given by

---

<sup>4</sup>Note that the decoupling limit in our notation,  $\alpha - \beta = \epsilon$  ( $\epsilon \rightarrow 0$ ), corresponds to  $\alpha - \beta = -\pi/2 + \epsilon$  ( $\epsilon \rightarrow 0$ ) in another notation [16].

$$\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}} = \frac{s+t}{f^2} C_3(\beta), \quad (4.43)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow hH)_{\text{cust}} = \frac{s}{f^2} C_4(\alpha, \beta), \quad (4.44)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow hA)_{\text{cust}} = -i \frac{s+2t}{3f^2} s_{\alpha-\beta} (c_{1221}^H - c_{1212}^H), \quad (4.45)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L H)_{\text{cust}} = 0, \quad (4.46)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L A)_{\text{cust}} = \frac{s}{f^2} C_3(\beta), \quad (4.47)$$

$$\mathcal{M}(Z_L Z_L \rightarrow W_L^+ H^-)_{\text{cust}} = \frac{s}{f^2} C_3(\beta), \quad (4.48)$$

$$\mathcal{M}(Z_L Z_L \rightarrow hH)_{\text{cust}} = \frac{s}{f^2} C_4(\alpha, \beta), \quad (4.49)$$

$$\mathcal{M}(Z_L Z_L \rightarrow hA)_{\text{cust}} = 0, \quad (4.50)$$

$$\mathcal{M}(Z_L Z_L \rightarrow Z_L H)_{\text{cust}} = 0, \quad (4.51)$$

$$\mathcal{M}(Z_L Z_L \rightarrow Z_L A)_{\text{cust}} = 0, \quad (4.52)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow H^+ h)_{\text{cust}} = -i \frac{s+2t}{3f^2} s_{\alpha-\beta} (c_{1221}^H - c_{1212}^H), \quad (4.53)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ H)_{\text{cust}} = 0, \quad (4.54)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ A)_{\text{cust}} = \frac{t}{f^2} C_3(\beta), \quad (4.55)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow H^+ Z_L)_{\text{cust}} = \frac{t}{f^2} C_3(\beta), \quad (4.56)$$

$$\mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ H^+)_{\text{cust}} = -\frac{s}{f^2} C_3(\beta), \quad (4.57)$$

where

$$\begin{aligned} C_3(\beta) = & \frac{1}{8} \left( -(2s_{2\beta} + s_{4\beta}) c_{1111}^H + 4(c_{2\beta} + c_{4\beta}) c_{1112}^H \right. \\ & + 2s_{4\beta} (c_{1122}^H + c_{1221}^H + c_{1212}^H) \\ & \left. + 4(c_{2\beta} - c_{4\beta}) c_{2221}^H + (2s_{2\beta} - s_{4\beta}) c_{2222}^H \right), \end{aligned} \quad (4.58)$$

$$\begin{aligned} C_4(\alpha, \beta) = & \frac{1}{8} \left( -((2s_{2\beta} + s_{4\beta}) c_{2(\alpha-\beta)} + (1 + 2c_{2\beta} + c_{4\beta}) s_{2(\alpha-\beta)}) c_{1111}^H \right. \\ & + 4((c_{2\beta} + c_{4\beta}) c_{2(\alpha-\beta)} - (s_{2\beta} + s_{4\beta}) s_{2(\alpha-\beta)}) c_{1112}^H \\ & + 2(s_{4\beta} c_{2(\alpha-\beta)} + (1 + c_{4\beta}) s_{2(\alpha-\beta)}) c_{1122}^H \\ & + 2(s_{4\beta} c_{2(\alpha-\beta)} - (1 - c_{4\beta}) s_{2(\alpha-\beta)}) (c_{1221}^H + c_{1212}^H) \\ & + 4((c_{2\beta} - c_{4\beta}) c_{2(\alpha-\beta)} - (s_{2\beta} - s_{4\beta}) s_{2(\alpha-\beta)}) c_{2221}^H \\ & \left. + ((2s_{2\beta} - s_{4\beta}) c_{2(\alpha-\beta)} - (1 - 2c_{2\beta} + c_{4\beta}) s_{2(\alpha-\beta)}) c_{2222}^H \right). \end{aligned} \quad (4.59)$$

As seen from above, Eqs. (4.43), (4.47), (4.48), (4.55), (4.56) and (4.57), are expressed by a common constant,  $C_3(\beta)$ , and Eqs. (4.44) and (4.49) are expressed by  $C_4(\alpha, \beta)$ . Among them, Eqs. (4.43) and (4.57) (Eqs. (4.47) and (4.55), Eqs. (4.48) and (4.56)) are related through the crossing symmetry. In the case without the custodial symmetry, the amplitudes of  $W_L^+ W_L^- \rightarrow W_L^+ H^-$  and  $W_L^+ W_L^+ \rightarrow W_L^+ H^+$  are different from the other four amplitudes because of  $c^T$  dependence. See Appendix C. Amplitudes in Eqs. (4.46) and (4.54) related by the crossing symmetry vanish in the case with the custodial invariance. We also obtain the equality between Eqs. (4.45) and (4.53). This equality is broken by the custodial symmetry breaking terms as shown in Appendix C.

The coefficient,  $C_4(\alpha, \beta)$ , is expanded in terms of  $\alpha - \beta$  as follows:

$$\begin{aligned} C_4(\alpha, \beta) = & C_3(\beta) \\ & + 2(\alpha - \beta) \left( (1 + 2c_{2\beta} + c_{4\beta})c_{1111}^H - 4(s_{2\beta} - s_{4\beta})c_{1112}^H + 2((1 + c_{4\beta})c_{1122}^H \right. \\ & \quad \left. - (1 - c_{4\beta})(c_{1221}^H + c_{1212}^H) - 4(s_{2\beta} - s_{4\beta})c_{2221}^H - (1 - 2c_{2\beta} + c_{4\beta})c_{2222}^H) \right) \\ & + \mathcal{O}((\alpha - \beta)^2), \end{aligned} \quad (4.60)$$

If we take the decoupling limit, all of nonzero amplitudes can be expressed by one parameter,  $C_3(\beta)$ . Hence, the following relations are satisfied:

$$\mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}} = \frac{s+t}{f^2} C_3(\beta), \quad (4.61)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L A)_{\text{cust}} = \mathcal{M}(W_L^+ W_L^- \rightarrow h H)_{\text{cust}} = \frac{s}{f^2} C_3(\beta). \quad (4.62)$$

Finally, amplitudes of double heavy Higgs boson production are listed:

$$\mathcal{M}(W_L^+ W_L^- \rightarrow H^+ H^-)_{\text{cust}} = \frac{s+t}{f^2} C_5(\beta) + \frac{t}{f^2} (c_{1221}^H - c_{1122}^H), \quad (4.63)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow H H)_{\text{cust}} = \frac{s}{f^2} C_6(\alpha, \beta), \quad (4.64)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow A A)_{\text{cust}} = \frac{s}{f^2} C_5(\beta), \quad (4.65)$$

$$\mathcal{M}(W_L^+ W_L^- \rightarrow H A)_{\text{cust}} = -i \frac{s+2t}{3f^2} c_{\alpha-\beta} (c_{1221}^H - c_{1212}^H), \quad (4.66)$$

$$\mathcal{M}(Z_L Z_L \rightarrow H^+ H^-)_{\text{cust}} = \frac{s}{f^2} C_5(\beta), \quad (4.67)$$

$$\mathcal{M}(Z_L Z_L \rightarrow H H)_{\text{cust}} = \frac{s}{f^2} C_6(\alpha, \beta), \quad (4.68)$$

$$\mathcal{M}(Z_L Z_L \rightarrow A A)_{\text{cust}} = \frac{s}{f^2} (c_{1122}^H - 3c_{1221}^T), \quad (4.69)$$

$$\mathcal{M}(Z_L Z_L \rightarrow H A)_{\text{cust}} = 0, \quad (4.70)$$



$$\mathcal{M}(W_L^+ Z_L \rightarrow H^+ H)_{\text{cust}} = -i \frac{s+2t}{3f^2} c_{\alpha-\beta} (c_{1221}^H - c_{1212}^H), \quad (4.71)$$

$$\mathcal{M}(W_L^+ Z_L \rightarrow H^+ A)_{\text{cust}} = \frac{t}{f^2} C_5(\beta) - \frac{s-t}{3f^2} c_{1221}^H + \frac{s+2t}{3f^2} c_{1212}^H - \frac{t}{f^2} c_{1122}^H + \frac{2s+t}{f^2} c_{1221}^T, \quad (4.72)$$

$$\mathcal{M}(W_L^+ W_L^+ \rightarrow H^+ H^+)_{\text{cust}} = -\frac{s}{f^2} C_5(\beta), \quad (4.73)$$

where

$$C_5(\beta) = \frac{1}{8} \left( (1 - c_{4\beta})(c_{1111}^H - 2c_{1221}^H - 2c_{1212}^H + c_{2222}^H) - 4s_{4\beta}(c_{1112}^H - c_{2221}^H) + 2(3 + c_{4\beta})c_{1122}^H \right), \quad (4.74)$$

$$C_6(\alpha, \beta) = \frac{1}{8} \left( (2(1 + c_{2\beta}) - (1 + 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} + (2s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})c_{1111}^H + 4(s_{2\beta} - (s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} - (c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})c_{1112}^H + 2(2 + (1 + c_{4\beta})c_{2(\alpha-\beta)} - s_{4\beta}s_{2(\alpha-\beta)})c_{1122}^H - 2((1 - c_{4\beta})c_{2(\alpha-\beta)} + s_{4\beta}s_{2(\alpha-\beta)})(c_{1221}^H + c_{1212}^H) + 4(s_{2\beta} - (s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} - (c_{2\beta} - c_{4\beta})s_{2(\alpha-\beta)})c_{2221}^H + (2(1 - c_{2\beta}) - (1 - 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (2s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})c_{2222}^H \right). \quad (4.75)$$

The amplitudes, Eqs. (4.65), (4.67) and (4.73), are identical up to overall sign, and the amplitudes, Eqs. (4.64) and (4.68), are identical. Among them, for the process,  $W_L^+ W_L^+ \rightarrow H^+ H^-$ ,  $c^T$  dependence appears when the custodial symmetry breaking terms exist. See Appendix C. Unlike previous cases, the processes,  $Z_L Z_L \rightarrow AA$  and  $W_L^+ Z_L \rightarrow H^+ A$ , have the  $c_{1221}^T$  dependence. Equation (4.66) is identical to Eq. (4.71) due to the custodial symmetry.

We present the power series expansion of  $\alpha - \beta$  for the coefficient,  $C_6(\alpha, \beta)$ , as follows:

$$C_6(\alpha, \beta) = C_5(\beta) + 2(\alpha - \beta) \left( (2s_{2\beta} + s_{4\beta})c_{1111}^H - 4(c_{2\beta} + c_{4\beta})c_{1112}^H - 2s_{4\beta}(c_{1122}^H + c_{1221}^H + c_{1212}^H) - 4(c_{2\beta} - c_{4\beta})c_{2221}^H - (2s_{2\beta} - s_{4\beta})c_{2222}^H \right) + \mathcal{O}((\alpha - \beta)^2). \quad (4.76)$$

The amplitude,  $\mathcal{M}(W_L^+ W_L^- \rightarrow HH)$ , becomes the same as  $\mathcal{M}(W_L^+ W_L^- \rightarrow AA)$  in the decoupling limit.

### 4.3 Cross sections and numerical results

Since we assume that masses of scalar bosons can be neglected, a cross section is written as

$$\sigma(V_1 V_2 \rightarrow X_1 X_2) = \frac{s}{32\pi f^4} \left( \frac{(2C_s - C_t)^2}{2} + \frac{C_t^2}{6} \right), \quad (4.77)$$

for an amplitude

$$\mathcal{M} = \frac{C_s s + C_t t}{f^2}. \quad (4.78)$$

If  $X_1 = X_2$ , the cross section must be divided by two. With the above formula, cross sections of vector boson fusion subsystems are given as follows:

$$\sigma(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}} = \frac{s}{32\pi f^4} \frac{2}{3} C_1(\beta)^2, \quad (4.79)$$

$$\sigma(W_L^+ W_L^- \rightarrow hh)_{\text{cust}} = \frac{s}{32\pi f^4} C_2(\alpha, \beta)^2, \quad (4.80)$$

$$\sigma(W_L^+ W_L^- \rightarrow Z_L Z_L)_{\text{cust}} = \frac{3}{2} \sigma(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (4.81)$$

$$\sigma(Z_L Z_L \rightarrow W_L^+ W_L^-)_{\text{cust}} = 3\sigma(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (4.82)$$

$$\sigma(Z_L Z_L \rightarrow hh)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow hh)_{\text{cust}}, \quad (4.83)$$

$$\sigma(W_L^+ Z_L \rightarrow W_L^+ Z_L)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (4.84)$$

$$\sigma(W_L^+ W_L^+ \rightarrow W_L^+ W_L^+)_{\text{cust}} = \frac{3}{2} \sigma(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (4.85)$$

$$\sigma(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}} = \frac{s}{32\pi f^4} \frac{2}{3} C_3(\beta)^2, \quad (4.86)$$

$$\sigma(W_L^+ W_L^- \rightarrow hH)_{\text{cust}} = \frac{s}{32\pi f^4} 2C_4(\alpha, \beta)^2, \quad (4.87)$$

$$\sigma(W_L^+ W_L^- \rightarrow hA)_{\text{cust}} = \frac{s}{32\pi f^4} \frac{2}{27} \sin^2(\alpha - \beta) (c_{1221}^H - c_{1212}^H)^2, \quad (4.88)$$

$$\sigma(W_L^+ W_L^- \rightarrow Z_L A)_{\text{cust}} = 3\sigma(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (4.89)$$

$$\sigma(Z_L Z_L \rightarrow W_L^+ H^-)_{\text{cust}} = 3\sigma(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (4.90)$$

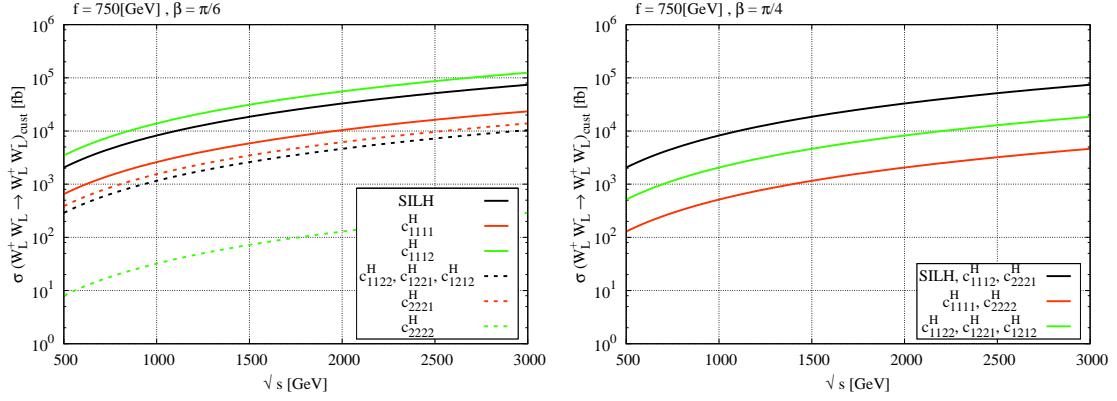
$$\sigma(Z_L Z_L \rightarrow hH)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow hH)_{\text{cust}}, \quad (4.91)$$

$$\sigma(W_L^+ Z_L \rightarrow H^+ h)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow hA)_{\text{cust}}, \quad (4.92)$$

$$\sigma(W_L^+ Z_L \rightarrow W_L^+ A)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (4.93)$$

$$\sigma(W_L^+ Z_L \rightarrow H^+ Z_L)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (4.94)$$

$$\sigma(W_L^+ W_L^+ \rightarrow W_L^+ H^+)_{\text{cust}} = 3\sigma(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (4.95)$$



**Figure 1.** Cross sections of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  for  $\beta = \pi/6$  (left) and  $\pi/4$  (right). The decay constant  $f$  is fixed as 750 GeV. The line of the SILH means the cross section of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  with  $\alpha = \beta = 0$ ,  $c_{1111}^H = 1$  and the others vanish. For the other lines, only one of coefficients is unity and the others are zero.

$$\sigma(W_L^+ W_L^- \rightarrow H^+ H^-)_{\text{cust}} = \frac{s}{32\pi f^4} \frac{2}{3} (C_5(\beta)^2 - C_5(\beta)(c_{1221}^H - c_{1122}^H) + (c_{1221}^H - c_{1122}^H)^2), \quad (4.96)$$

$$\sigma(W_L^+ W_L^- \rightarrow HH)_{\text{cust}} = \frac{s}{32\pi f^4} C_6(\alpha, \beta)^2, \quad (4.97)$$

$$\sigma(W_L^+ W_L^- \rightarrow AA)_{\text{cust}} = \frac{s}{32\pi f^4} C_5(\beta)^2, \quad (4.98)$$

$$\sigma(W_L^+ W_L^- \rightarrow HA)_{\text{cust}} = \frac{s}{32\pi f^4} \frac{2}{27} \cos^2(\alpha - \beta) (c_{1221}^H - c_{1212}^H)^2, \quad (4.99)$$

$$\sigma(Z_L Z_L \rightarrow H^+ H^-)_{\text{cust}} = 2\sigma(W_L^+ W_L^- \rightarrow AA)_{\text{cust}}, \quad (4.100)$$

$$\sigma(Z_L Z_L \rightarrow HH)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow HH)_{\text{cust}}, \quad (4.101)$$

$$\sigma(Z_L Z_L \rightarrow AA)_{\text{cust}} = \frac{s}{32\pi f^4} (c_{1122}^H - 3c_{1221}^T)^2, \quad (4.102)$$

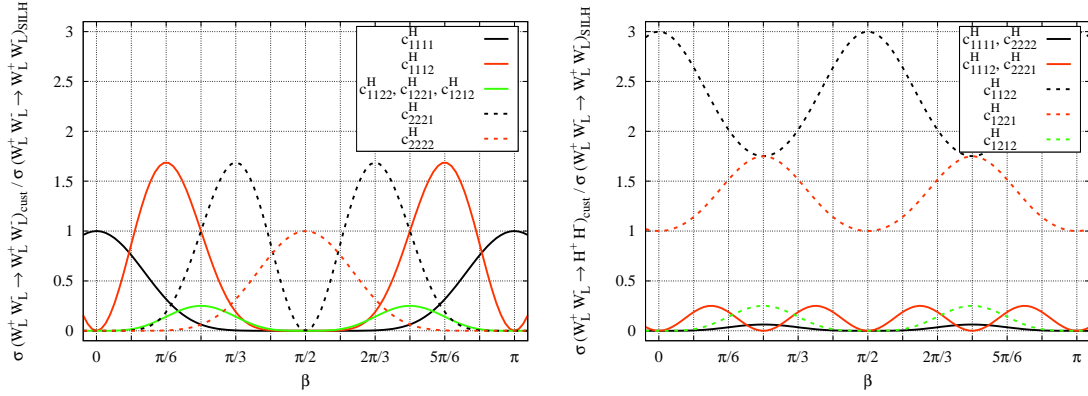
$$\sigma(W_L^+ Z_L \rightarrow H^+ H)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow HA)_{\text{cust}}, \quad (4.103)$$

$$\begin{aligned} \sigma(W_L^+ Z_L \rightarrow H^+ A)_{\text{cust}} = & \frac{s}{32\pi f^4} \frac{1}{2} \left( (C_5(\beta) - c_{1122}^H + c_{1221}^H - 3c_{1221}^T)^2 \right. \\ & \left. + \frac{1}{3} \left( C_5(\beta) + \frac{c_{1221}^H + 2c_{1212}^H}{3} - c_{1122}^H + c_{1221}^T \right)^2 \right), \end{aligned} \quad (4.104)$$

$$\sigma(W_L^+ W_L^+ \rightarrow H^+ H^+)_{\text{cust}} = \sigma(W_L^+ W_L^- \rightarrow AA)_{\text{cust}}. \quad (4.105)$$

In these cross sections, we omitted processes whose amplitudes are zero. Some of cross sections are proportional to others even if their amplitudes are not. Notice that since we still have ten independent cross sections, all of parameters,  $c^{H,T}/f^2$ ,  $\alpha$  and  $\beta$ , can be fixed by measuring these processes.

Cross sections depend on eight coefficients and two angles. In the following figures, one of coefficients is turned on and the others are turned off. Coefficients turned on are set



**Figure 2.** The  $\beta$  dependences of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  (left) and  $W_L^+ W_L^- \rightarrow H^+ H^-$  (right). These cross sections are given in the unit of the  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  cross section with the SILH parameters.

as unity. The parameter that  $\alpha = \beta = 0$  and only  $c_{1111}^H \neq 0$  reproduces the SILH, hence we call this case the SILH. The decay constant of the NL $\Sigma$ M,  $f$ , is set to be 750 GeV.

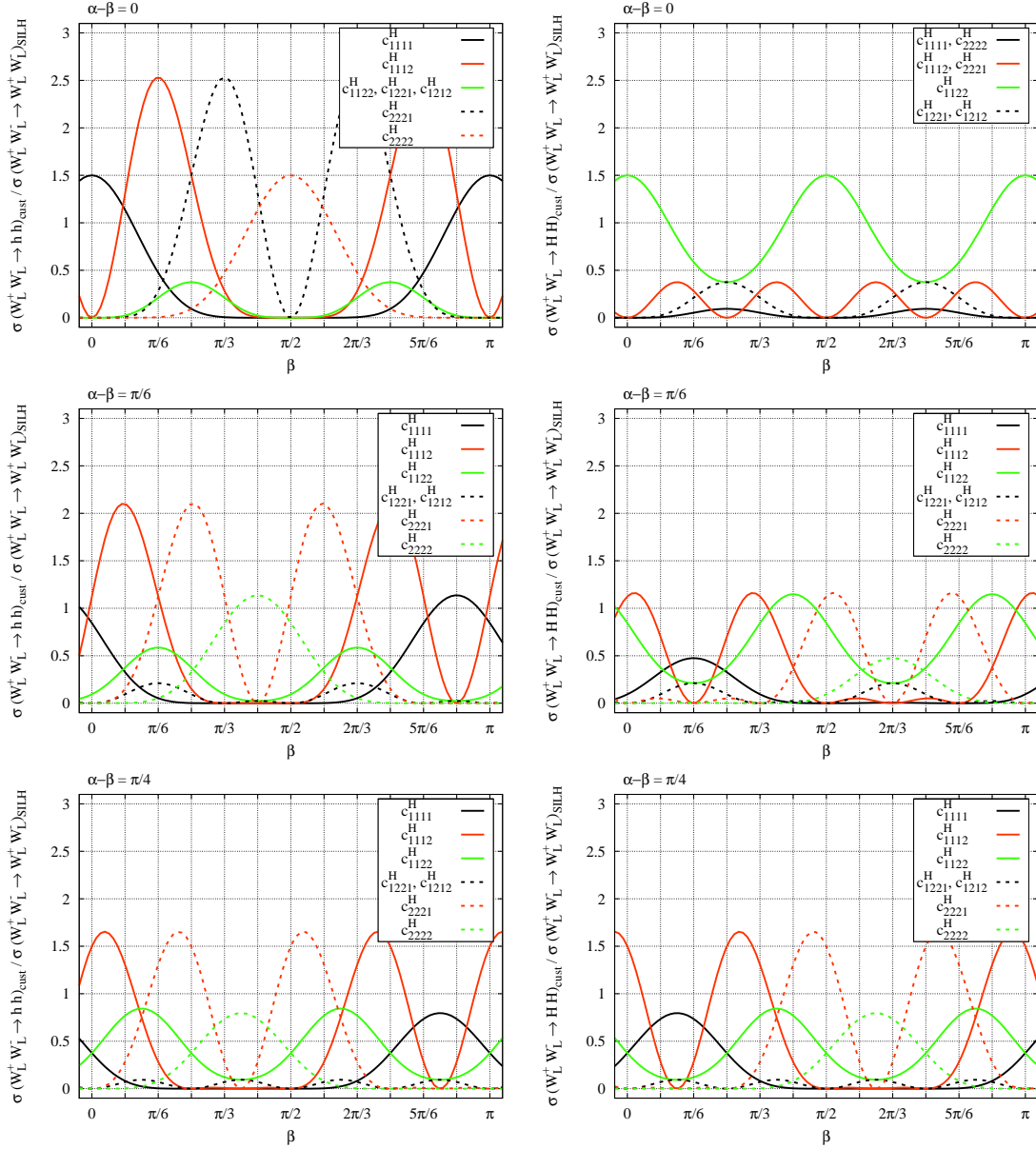
In Fig. 1, cross sections of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  are presented, where  $\beta$  is chosen as  $\pi/6$  or  $\pi/4$  and, for each line, only one of coefficients is turned on. Since all lines are linear functions of the squared invariant mass,  $s$ , they are parallel to each other.

In Ref. [9] it is shown that focusing on the central region,  $-1/2 < \cos \theta < 1/2$ , helps us discriminate the derivative interactions from the other contributions in high energy region. Without restriction to the central region, the cross section of the longitudinal mode production is much smaller than that of the transverse mode in the SM ( $\sim 2 \times 10^6$  fb)<sup>5</sup>. Further investigation with polarization measurement proposed by Ref. [8] could be useful to distinguish the longitudinal mode from the transverse mode at the lower energy region. Cross sections of the central region are given in Appendix E.

Cross sections of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  and  $W_L^+ W_L^- \rightarrow H^+ H^-$  as functions of  $\beta$  are shown in Fig. 2. Each line corresponds to the case where one of coefficients is nonzero. Cross sections are presented in the unit of the  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  cross section with the SILH condition. For  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ , the largest ratios given by  $c_{1112}^H$  and  $c_{2221}^H$  are 27/16. Peaks of small bumps given by  $c_{1122}^H$ ,  $c_{1221}^H$  and  $c_{1212}^H$  are one fourth comparing to the SILH cross section. Absence of  $c_{1111}^H$  and  $c_{2222}^H$ , respectively, erases the cross section at  $\beta = 0$  and  $\pi/2$ . For  $W_L^+ W_L^- \rightarrow H^+ H^-$ ,  $c_{1122}^H$  and  $c_{1221}^H$  generate larger cross section than that given by  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  in the SILH. Contributions of  $c_{1122}^H$  and  $c_{1221}^H$  do not vanish for any  $\beta$ . In summary, cross sections receive large contribution from  $c_{1112}^H$  and  $c_{2221}^H$  for  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  and from  $c_{1122}^H$  and  $c_{1221}^H$  for  $W_L^+ W_L^- \rightarrow H^+ H^-$ . On the other hand, if only  $c_{1212}^H$  appears, the production cross sections of both processes are much smaller than the  $W$  boson pair production cross section in the SM.

Figure 3 shows  $\beta$  dependences of  $W_L^+ W_L^- \rightarrow hh$  and  $W_L^+ W_L^- \rightarrow HH$  for  $\alpha = \beta = 0$ ,

<sup>5</sup> Our SILH line ( $f = 750$  GeV,  $\alpha = \beta = 0$  and  $c_{1111}^H = 1$ ) corresponds to the line of  $a^2 \sim 0.9$  in terms of Ref. [9].



**Figure 3.** The  $\beta$  dependences of  $W_L^+ W_L^- \rightarrow hh$  (left) and  $W_L^+ W_L^- \rightarrow HH$  (right). Each row means  $\alpha - \beta = 0, \pi/6$  and  $\pi/4$ . These cross sections are given in the unit of the  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  cross section with the SILH parameters.

$\pi/6$  and  $\pi/4$ <sup>6</sup>. Their cross sections are divided by the cross section of  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  in the SILH. These cross sections can exceed the SM  $WW \rightarrow hh$  production cross section ( $\sim 5 \times 10^4$  fb) in a few TeV region without studying on the central region. By analysis in the central region, the contributions of derivative interactions can be distinguished from the other effects even in the region lower than 1 TeV.

<sup>6</sup> The cross section of  $W_L^+ W_L^- \rightarrow hh$  with the SILH case ( $f = 750$  GeV,  $\alpha = \beta = 0$  and  $c_{1111}^H = 1$ ) corresponds to that with the line of  $a^2 - b \sim 0.9$  in terms of Ref. [9].

In summary, we found that the longitudinal mode of  $W$  boson and the charged Higgs boson possessed quite different parameter dependences. Because the production cross section of  $W^+W^- \rightarrow hh$  is smaller than that of  $W^+W^- \rightarrow W^+W^-$  by 2 orders of magnitude in the SM,  $W_L^+W_L^- \rightarrow hh$  is the promising process to see effects of derivative interactions. There are several possibilities to discriminate the NG 2HDM from the SILH depending on parameters: the decay constant, coefficients, angles and mass spectra of heavy Higgs bosons. For example, since the cross section of  $W_L^+W_L^- \rightarrow hh$  is three halves larger than that of  $W_L^+W_L^- \rightarrow W_L^+W_L^-$  in the SILH, the difference from the SILH could be easily found if  $W_L^+W_L^- \rightarrow W_L^+W_L^-$  is observed. In the case that additional Higgs bosons are light, other processes may be promising to discriminate this model from the SILH.

## 5 Conclusions

We have extended the treatment of the dimension-six derivative interactions in the SILH model to the case of NHDM. The operators are phenomenologically important because they determine high energy behavior of the scattering amplitudes of the longitudinal gauge bosons and the Higgs bosons.

In the NL $\Sigma$ M, the information of the global symmetry breaking determines the form of the operators, and hence the derivative interactions are governed by the structure constant. Since the  $N$  Higgs doublets can be embedded in the NG fields as the  $SO(4N)$  multiplet, the derivative interactions can be expressed using the  $SO(4N)$  generators. Using the bidoublet notation, it is easy to impose the  $SU(2)_L \times U(1)_Y$  to the interactions. As a consequence, the number of the derivative interactions,  $(3/2)N^2(N^2 + 1)$  real DOF and  $(1/2)N^2(3N^2 - 1)$  imaginary DOF in the general NHDM, is reduced to  $(1/2)N^2(N^2 + 3)$  real DOF and  $(1/2)N^2(N^2 - 1)$  imaginary DOF due to the nature of a strongly interacting dynamics.

We have then applied these results to the 2HDM. By the phenomenological requirement, we impose the manifest custodial invariance. We have calculated the scattering amplitudes of the longitudinal gauge bosons and the Higgs bosons by the dimension-six derivative interactions. We have derived various relations among the scattering amplitudes, and clarified the differences between the custodial symmetry violating case and preserving case. The amplitudes including only the SM particles violate the simple relations found in the SILH model. These relations are recovered in the decoupling limit,  $\alpha - \beta = 0$ . In other words, the parameter  $\alpha - \beta$  is a key parameter to distinguish the composite 2HDM from the SILH model. The high energy behavior of the scattering amplitudes can provide an alternative way to study the Higgs sector to the Higgs coupling measurements through the Higgs boson production and decay processes. We have obtained several relations among the amplitudes involving heavy Higgs bosons, which provides us clues to the structure of the 2HDM described by the NL $\Sigma$ M. Pair production cross sections of longitudinal gauge bosons and Higgs bosons due to dimension-six operators also have been calculated. We have found that  $W_L^+W_L^- \rightarrow hh$  is a promising process to see the effect of derivative interactions. Cross sections of other processes may be large enough to be observed, so that their observations are useful to distinguish this model from the SILH.

Even if the Higgs boson is discovered, it is not easy to reveal the structure of the Higgs sector. By investigation of the strongly interacting NHDM, we show that precise measurements of the longitudinal gauge boson scatterings can give us hints for physics beyond the SM; how many the Higgs doublets exist, whether the fundamental interactions are strong or weak, etc. These phenomena are important physics targets in collider experiments at the LHC and LC.

## Acknowledgments

The authors would like to thank P. Posch for useful discussions. The work of Y.O. is supported in part by the Grant-in-Aid for Science Research, Japan Society for the Promotion of Science (JSPS), No.20244037 and No.22244031. The work of Y.Y. is supported in part by the Grant-in-Aid for Science Research, Japan Society for the Promotion of Science (JSPS), No.22.3834.

## A Generators of the $SO(4N)$

In order to respect the  $SO(4) \simeq SU(2)_L \times SU(2)_R$  symmetry, it is convenient to classify the generators of the  $SO(4N)$  into the irreducible representation of the  $SO(4)$ . We can write  $2N(4N - 1)$  generators in terms of the Kronecker delta:

$$\left(T_{(i,j)}^{ab}\right)_{cd} = -\frac{i}{2} \left( \delta^{a+4(i-1),c} \delta^{b+4(j-1),d} - \delta^{a+4(i-1),d} \delta^{b+4(j-1),c} \right), \quad (\text{A.1})$$

where  $i, j \in \{1, \dots, N\}$  ( $i \leq j$ ),  $a, b \in \{1, \dots, 4\}$  and  $c, d \in \{1, \dots, 4N\}$ . Namely,  $i$  and  $j$  mean the indices of  $4 \times 4$  blocks and  $a$  and  $b$  stand for components in each blocks.

The generators can be classified as follows:

$$T_{(i,i)}^{L1} = -T_{(i,i)}^{14} + T_{(i,i)}^{23}, \quad (\text{A.2})$$

$$T_{(i,i)}^{L2} = T_{(i,i)}^{13} + T_{(i,i)}^{24}, \quad (\text{A.3})$$

$$T_{(i,i)}^{L3} = -T_{(i,i)}^{12} + T_{(i,i)}^{21}, \quad (\text{A.4})$$

$$T_{(i,i)}^{R1} = -T_{(i,i)}^{14} - T_{(i,i)}^{23}, \quad (\text{A.5})$$

$$T_{(i,i)}^{R2} = T_{(i,i)}^{13} - T_{(i,i)}^{24}, \quad (\text{A.6})$$

$$T_{(i,i)}^{R3} = -T_{(i,i)}^{12} - T_{(i,i)}^{21}, \quad (\text{A.7})$$

$$T_{(i,j)}^{L1} = -T_{(i,j)}^{14} + T_{(i,j)}^{23} - T_{(i,j)}^{32} + T_{(i,j)}^{41}, \quad (\text{A.8})$$

$$T_{(i,j)}^{L2} = T_{(i,j)}^{13} + T_{(i,j)}^{24} - T_{(i,j)}^{31} - T_{(i,j)}^{42}, \quad (\text{A.9})$$

$$T_{(i,j)}^{L3} = -T_{(i,j)}^{12} + T_{(i,j)}^{21} + T_{(i,j)}^{34} - T_{(i,j)}^{43}, \quad (\text{A.10})$$

$$T_{(i,j)}^{R1} = -T_{(i,j)}^{14} - T_{(i,j)}^{23} + T_{(i,j)}^{32} + T_{(i,j)}^{41}, \quad (\text{A.11})$$

$$T_{(i,j)}^{R2} = T_{(i,j)}^{13} - T_{(i,j)}^{24} + T_{(i,j)}^{31} - T_{(i,j)}^{42}, \quad (\text{A.12})$$

$$T_{(i,j)}^{R3} = -T_{(i,j)}^{12} + T_{(i,j)}^{21} - T_{(i,j)}^{34} + T_{(i,j)}^{43}, \quad (\text{A.13})$$

$$U_{(i,j)} = T_{(i,j)}^{11} + T_{(i,j)}^{22} + T_{(i,j)}^{33} + T_{(i,j)}^{44}, \quad (\text{A.14})$$

$$S_{(i,j)}^{11} = T_{(i,j)}^{11} - T_{(i,j)}^{22} - T_{(i,j)}^{33} + T_{(i,j)}^{44}, \quad (\text{A.15})$$

$$S_{(i,j)}^{12} = -T_{(i,j)}^{12} - T_{(i,j)}^{21} + T_{(i,j)}^{34} + T_{(i,j)}^{43}, \quad (\text{A.16})$$

$$S_{(i,j)}^{13} = T_{(i,j)}^{13} + T_{(i,j)}^{24} + T_{(i,j)}^{31} + T_{(i,j)}^{42}, \quad (\text{A.17})$$

$$S_{(i,j)}^{21} = -T_{(i,j)}^{12} - T_{(i,j)}^{21} - T_{(i,j)}^{34} - T_{(i,j)}^{43}, \quad (\text{A.18})$$

$$S_{(i,j)}^{22} = -T_{(i,j)}^{11} + T_{(i,j)}^{22} - T_{(i,j)}^{33} + T_{(i,j)}^{44}, \quad (\text{A.19})$$

$$S_{(i,j)}^{23} = T_{(i,j)}^{14} - T_{(i,j)}^{23} - T_{(i,j)}^{32} + T_{(i,j)}^{41}, \quad (\text{A.20})$$

$$S_{(i,j)}^{31} = -T_{(i,j)}^{13} + T_{(i,j)}^{24} - T_{(i,j)}^{31} + T_{(i,j)}^{42}, \quad (\text{A.21})$$

$$S_{(i,j)}^{32} = T_{(i,j)}^{14} + T_{(i,j)}^{23} + T_{(i,j)}^{32} + T_{(i,j)}^{41}, \quad (\text{A.22})$$

$$S_{(i,j)}^{33} = T_{(i,j)}^{11} + T_{(i,j)}^{22} - T_{(i,j)}^{33} - T_{(i,j)}^{44}, \quad (\text{A.23})$$

where  $T_{(i,j)}^{L\alpha}$ ,  $T_{(i,j)}^{R\beta}$ ,  $U_{(i,j)}$  and  $S_{(i,j)}^{\alpha\beta}$  are  $(\mathbf{3}, \mathbf{1})$ ,  $(\mathbf{1}, \mathbf{3})$ ,  $(\mathbf{1}, \mathbf{1})$  and  $(\mathbf{3}, \mathbf{3})$  representations of  $SU(2)_L \times SU(2)_R$ , respectively. Normalization depends on whether the generator is in diagonal block or off-diagonal block. According to the above, all kinds of generators appear for  $N = 2$ . Therefore, as an example, we show the generators of the  $SO(4)$  and the generators,  $(\mathbf{1}, \mathbf{1})$  and  $(\mathbf{3}, \mathbf{3})$ , in the  $SO(8)$  below:

$$T^{L1} = \frac{i}{2} \begin{pmatrix} & & & 1 \\ & & -1 & \\ & 1 & & \\ -1 & & & \end{pmatrix}, \quad T^{L2} = \frac{i}{2} \begin{pmatrix} & -1 & & \\ & & -1 & \\ 1 & & & \\ & 1 & & \end{pmatrix}, \quad T^{L3} = \frac{i}{2} \begin{pmatrix} & & 1 & \\ -1 & & & \\ & & & -1 \\ & & 1 & \end{pmatrix} \quad (\text{A.24})$$

$$T^{R1} = \frac{i}{2} \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}, \quad T^{R2} = \frac{i}{2} \begin{pmatrix} & 1 & & \\ & & -1 & \\ -1 & & & \\ & 1 & & \end{pmatrix}, \quad T^{R3} = \frac{i}{2} \begin{pmatrix} & & 1 & \\ -1 & & & \\ & & & 1 \\ & & -1 & \end{pmatrix} \quad (\text{A.25})$$

$$U_{(1,2)} = \frac{i}{2} \begin{pmatrix} \mathbf{0}_4 & -\mathbf{1}_4 \\ \mathbf{1}_4 & \mathbf{0}_4 \end{pmatrix}, \quad S_{(1,2)}^{\alpha\beta} = \frac{i}{2} \begin{pmatrix} \mathbf{0}_4 & S^{\alpha\beta} \\ -S^{\alpha\beta} & \mathbf{0}_4 \end{pmatrix}, \quad (\text{A.26})$$



$$S^{11} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}, \quad S^{12} = \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad S^{13} = \begin{pmatrix} & & -1 & \\ & & & -1 \\ -1 & & & \\ & -1 & & \end{pmatrix}, \quad (\text{A.27})$$

$$S^{21} = \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & 1 & \\ & & & 1 \end{pmatrix}, \quad S^{22} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}, \quad S^{23} = \begin{pmatrix} & & -1 & \\ & & 1 & \\ & 1 & & \\ -1 & & & \end{pmatrix}, \quad (\text{A.28})$$

$$S^{31} = \begin{pmatrix} & & 1 & \\ & & & -1 \\ 1 & & & \\ & -1 & & \end{pmatrix}, \quad S^{32} = \begin{pmatrix} & & & -1 \\ & & -1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}, \quad S^{33} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}, \quad (\text{A.29})$$

where blanks are filled by zero and  $\mathbf{0}_4$  and  $\mathbf{1}_4$  are the empty matrix and the unit matrix of four dimensions.

## B Bidoublet notation

The bidoublet notation is useful to see the  $SU(2)_L \times SU(2)_R \simeq SO(4)$  symmetry. We use the following  $4 \times 4$  matrix as bidoublet:

$$\Phi_{ii} = \begin{pmatrix} i\sigma^2 H_i^* & H_i \end{pmatrix}. \quad (\text{B.1})$$

Under the  $SU(2)_L \times SU(2)_R$  symmetry, the transformation law of the bidoublet is

$$\Phi_{ii} \rightarrow L \Phi_{ii} R^\dagger, \quad (\text{B.2})$$

where  $L \in SU(2)_L$  and  $R \in SU(2)_R$ .

The correspondences between the  $SU(2)_L$  doublets and the bidoublets are

$$H_i^\dagger H_j = \frac{1}{2} \text{Tr} \left[ \Phi_{ii}^\dagger \Phi_{jj} - \sigma^3 \Phi_{ii}^\dagger \Phi_{jj} \right], \quad (\text{B.3})$$

$$H_j^\dagger H_i = \frac{1}{2} \text{Tr} \left[ \Phi_{ii}^\dagger \Phi_{jj} + \sigma^3 \Phi_{ii}^\dagger \Phi_{jj} \right], \quad (\text{B.4})$$

$$\partial_\mu H_i^\dagger \partial_\nu H_j = \frac{1}{2} \text{Tr} \left[ \partial_\mu \Phi_{ii}^\dagger \partial_\nu \Phi_{jj} - \sigma^3 \partial_\mu \Phi_{ii}^\dagger \partial_\nu \Phi_{jj} \right], \quad (\text{B.5})$$

$$\partial_\mu H_j^\dagger \partial_\nu H_i = \frac{1}{2} \text{Tr} \left[ \partial_\nu \Phi_{ii}^\dagger \partial_\mu \Phi_{jj} + \sigma^3 \partial_\nu \Phi_{ii}^\dagger \partial_\mu \Phi_{jj} \right], \quad (\text{B.6})$$

$$H_i^\dagger \overleftrightarrow{\partial}_\mu H_j = \frac{1}{2} \text{Tr} \left[ \Phi_{ii}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{jj} - \sigma^3 \Phi_{ii}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{jj} \right], \quad (\text{B.7})$$

$$H_j^\dagger \overleftrightarrow{\partial}_\mu H_i = \frac{1}{2} \text{Tr} \left[ \Phi_{ii}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{jj} + \sigma^3 \Phi_{ii}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{jj} \right], \quad (\text{B.8})$$

where the following relations are used:

$$\text{Tr} [\Phi_{ii}^\dagger \Phi_{jj}] = \text{Tr} [\Phi_{jj}^\dagger \Phi_{ii}], \quad (\text{B.9})$$

$$\text{Tr} [\sigma^3 \Phi_{ii}^\dagger \Phi_{jj}] = -\text{Tr} [\sigma^3 \Phi_{jj}^\dagger \Phi_{ii}], \quad (\text{B.10})$$

$$\text{Tr} [\Phi_{ii}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{jj}] = -\text{Tr} [\Phi_{jj}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{ii}], \quad (\text{B.11})$$

$$\text{Tr} [\sigma^3 \Phi_{ii}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{jj}] = \text{Tr} [\sigma^3 \Phi_{jj}^\dagger \overleftrightarrow{\partial}_\mu \Phi_{ii}]. \quad (\text{B.12})$$

Any potential terms and dimension-six derivative interactions can be described using the relations. In the potential and the derivative interaction, terms including  $\sigma^3$  violate the  $SO(4)$  symmetry.

## C Amplitudes for 2HDM without the $SO(4)$ symmetry

Amplitudes of the longitudinal modes and the Higgs bosons generated by the dimension-six derivative interactions are displayed below for 2HDM. We here consider the amplitudes based on the following Lagrangian:

$$\begin{aligned} \mathcal{L}_{2\text{HDM}}^6 = & \frac{c_{1111}^H}{2f^2} O_{1111}^H + \frac{c_{1112}^H}{f^2} (O_{1112}^H + O_{1121}^H) + \frac{c_{1122}^H}{f^2} O_{1122}^H + \frac{c_{1221}^H}{f^2} O_{1221}^H \\ & + \frac{c_{1212}^H}{2f^2} (O_{1212}^H + O_{2121}^H) + \frac{c_{2221}^H}{f^2} (O_{2212}^H + O_{2221}^H) + \frac{c_{2222}^H}{2f^2} O_{2222}^H \\ & + \frac{c_{1111}^T}{2f^2} O_{1111}^T + \frac{c_{1112}^T}{f^2} (O_{1112}^T + O_{1121}^T) + \frac{c_{1122}^T}{f^2} O_{1122}^T + \frac{c_{1221}^T}{f^2} O_{1221}^T \\ & + \frac{c_{1212}^T}{2f^2} (O_{1212}^T + O_{2121}^T) + \frac{c_{2221}^T}{f^2} (O_{2212}^T + O_{2221}^T) + \frac{c_{2222}^T}{2f^2} O_{2222}^T, \end{aligned} \quad (\text{C.1})$$

where, for simplicity, we assume all coefficients are real and the spontaneous  $CP$  violation is avoided. The Lagrangian apparently violates the custodial symmetry at the tree level due to the contributions of operators  $\mathcal{O}^T$ . In the following, initial states,  $V_1$  and  $V_2$ , are the longitudinal modes of massive gauge bosons,  $W_L^\pm$  and  $Z_L$ . The definitions of the Mandelstam variables are given by Eqs. (4.26), (4.27) and (4.28). In order to clarify the difference from the custodial invariant case, we also show the results in which amplitudes are decomposed into the custodial invariant part,  $\mathcal{M}_{\text{cust}}$ , given in Sec. 4.2 and the custodial symmetry violating part.

Firstly, amplitudes producing the SM particles are displayed i.e.  $V_1 V_2 \rightarrow X_1 X_2$  ( $X_1, X_2 \in$

$$\{W_L^\pm, Z_L, h\}:$$

$$\begin{aligned} \mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &= \frac{s+t}{8f^2} \left( (3+4c_{2\beta}+c_{4\beta})(c_{1111}^H+3c_{1111}^T)+4(2s_{2\beta}+s_{4\beta})(c_{1112}^H+3c_{1112}^T) \right. \\ &\quad + 2(1-c_{4\beta})(c_{1122}^H+c_{1221}^H+c_{1212}^H+3(c_{1122}^T+c_{1221}^T+c_{1212}^T)) \\ &\quad \left. + 4(2s_{2\beta}-s_{4\beta})(c_{2221}^H+3c_{2221}^T)+(3-4c_{2\beta}+c_{4\beta})(c_{2222}^H+3c_{2222}^T) \right) \end{aligned} \quad (\text{C.2})$$

$$\begin{aligned} &= \mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}} \\ &\quad + \frac{3(s+t)}{8f^2} \left( (3+4c_{2\beta}+c_{4\beta})c_{1111}^T+4(2s_{2\beta}+s_{4\beta})c_{1112}^T \right. \\ &\quad + 2(1-c_{4\beta})(c_{1122}^T+c_{1221}^T+c_{1212}^T) \\ &\quad \left. + 4(2s_{2\beta}-s_{4\beta})c_{2221}^T+(3-4c_{2\beta}+c_{4\beta})c_{2222}^T \right), \end{aligned} \quad (\text{C.3})$$

$$\begin{aligned} \mathcal{M}(W_L^+ W_L^- \rightarrow hh) &= \frac{s}{8f^2} \left( (2+2c_{2\beta}+(1+2c_{2\beta}+c_{4\beta})c_{2(\alpha-\beta)}-(2s_{2\beta}+s_{4\beta})s_{2(\alpha-\beta)})c_{1111}^H \right. \\ &\quad + 4(s_{2\beta}+(s_{2\beta}+s_{4\beta})c_{2(\alpha-\beta)}+(c_{2\beta}+c_{4\beta})s_{2(\alpha-\beta)})c_{1112}^H \\ &\quad + 2(2-(1+c_{4\beta})c_{2(\alpha-\beta)}+s_{4\beta}s_{2(\alpha-\beta)})c_{1122}^H \\ &\quad + 2((1-c_{4\beta})c_{2(\alpha-\beta)}+s_{4\beta}s_{2(\alpha-\beta)})(c_{1221}^H+c_{1212}^H) \\ &\quad + 4(s_{2\beta}+(s_{2\beta}-s_{4\beta})c_{2(\alpha-\beta)}+(c_{2\beta}-c_{4\beta})s_{2(\alpha-\beta)})c_{2221}^H \\ &\quad \left. + (2-2c_{2\beta}+(1-2c_{2\beta}+c_{4\beta})c_{2(\alpha-\beta)}+(2s_{2\beta}-s_{4\beta})s_{2(\alpha-\beta)})c_{2222}^H \right) \end{aligned} \quad (\text{C.4})$$

$$= \mathcal{M}(W_L^+ W_L^- \rightarrow hh)_{\text{cust}}, \quad (\text{C.5})$$

$$\begin{aligned} \mathcal{M}(W_L^+ W_L^- \rightarrow Z_L Z_L) &= \frac{s}{8f^2} \left( (3+4c_{2\beta}+c_{4\beta})c_{1111}^H+4(2s_{2\beta}+s_{4\beta})c_{1112}^H \right. \\ &\quad + 2(1-c_{4\beta})(c_{1122}^H+c_{1221}^H+c_{1212}^H) \\ &\quad \left. + 4(2s_{2\beta}-s_{4\beta})c_{2221}^H+(3-4c_{2\beta}+c_{4\beta})c_{2222}^H \right) \end{aligned} \quad (\text{C.6})$$

$$= \mathcal{M}(W_L^+ W_L^- \rightarrow Z_L Z_L)_{\text{cust}}, \quad (\text{C.7})$$

$$\begin{aligned} \mathcal{M}(W_L^+ W_L^- \rightarrow hZ_L) &= i \frac{s+2t}{8f^2} \left( ((3+4c_{2\beta}+c_{4\beta})c_{\alpha-\beta}-(2s_{2\beta}+s_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\ &\quad + 4((2s_{2\beta}+s_{4\beta})c_{\alpha-\beta}+(c_{2\beta}+c_{4\beta})s_{\alpha-\beta})c_{1112}^T \\ &\quad + 2((1-c_{4\beta})c_{\alpha-\beta}+s_{4\beta}s_{\alpha-\beta})(c_{1122}^T+c_{1221}^T+c_{1212}^T) \\ &\quad + 4((2s_{2\beta}-s_{4\beta})c_{\alpha-\beta}+(c_{2\beta}-c_{4\beta})s_{\alpha-\beta})c_{2221}^T \\ &\quad \left. + ((3-4c_{2\beta}+c_{4\beta})c_{\alpha-\beta}+(2s_{2\beta}-s_{4\beta})s_{\alpha-\beta})c_{2222}^T \right), \end{aligned} \quad (\text{C.8})$$

$$\begin{aligned}\mathcal{M}(Z_L Z_L \rightarrow W_L^+ W_L^-) &= \frac{s}{8f^2} \left( (3 + 4c_{2\beta} + c_{4\beta})c_{1111}^H + 4(2s_{2\beta} + s_{4\beta})c_{1112}^H \right. \\ &\quad + 2(1 - c_{4\beta})(c_{1122}^H + c_{1221}^H + c_{1212}^H) \\ &\quad \left. + 4(2s_{2\beta} - s_{4\beta})c_{2221}^H + (3 - 4c_{2\beta} + c_{4\beta})c_{2222}^H \right) \quad (\text{C.9})\end{aligned}$$

$$= \mathcal{M}(Z_L Z_L \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (\text{C.10})$$

$$\begin{aligned}\mathcal{M}(Z_L Z_L \rightarrow hh) &= \frac{s}{8f^2} \left( (2 + 2c_{2\beta} + (1 + 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (2s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})(c_{1111}^H + 3c_{1111}^T) \right. \\ &\quad + 4(s_{2\beta} + (s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} + (c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})(c_{1112}^H + 3c_{1112}^T) \\ &\quad + (1 + (1 - 2c_{4\beta})c_{2(\alpha-\beta)} + 2s_{4\beta}s_{2(\alpha-\beta)})(c_{1122}^H + c_{1221}^H + 3(c_{1122}^T + c_{1221}^T)) \\ &\quad + 3(1 - c_{2(\alpha-\beta)})(c_{1122}^H - c_{1221}^H - c_{1122}^T + c_{1221}^T) \\ &\quad + 2(1 - c_{4\beta}c_{2(\alpha-\beta)} + s_{4\beta}s_{2(\alpha-\beta)})(c_{1212}^H + 3c_{1212}^T) \\ &\quad + 4(s_{2\beta} + (s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} + (c_{2\beta} - c_{4\beta})s_{2(\alpha-\beta)})(c_{2221}^H + 3c_{2221}^T) \\ &\quad \left. + (2 - 2c_{2\beta} + (1 - 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} + (2s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})(c_{2222}^H + 3c_{2222}^T) \right) \quad (\text{C.11})\end{aligned}$$

$$\begin{aligned}&= \mathcal{M}(Z_L Z_L \rightarrow hh)_{\text{cust}} \\ &\quad + \frac{3s}{8f^2} \left( (2 + 2c_{2\beta} + (1 + 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (2s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})c_{1111}^T \right. \\ &\quad + 4(s_{2\beta} + (s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} + (c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})c_{1112}^T \\ &\quad + (c_{2(\alpha-\beta)}(1 - c_{4\beta}) + s_{2(\alpha-\beta)}s_{4\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\ &\quad + 4(s_{2\beta} + (s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} + (c_{2\beta} - c_{4\beta})s_{2(\alpha-\beta)})c_{2221}^T \\ &\quad \left. + (2 - 2c_{2\beta} + (1 - 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} + (2s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})c_{2222}^T \right) \\ &\quad + \frac{s}{4f^2} (1 - c_{2(\alpha-\beta)}) \left( 3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H) \right), \quad (\text{C.12})\end{aligned}$$

$$\mathcal{M}(Z_L Z_L \rightarrow Z_L Z_L) = 0, \quad (\text{C.13})$$

$$\mathcal{M}(Z_L Z_L \rightarrow h Z_L) = 0, \quad (\text{C.14})$$

$$\begin{aligned}
\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ h) = & -i \frac{2s+t}{8f^2} \left( ((3+4c_{2\beta}+c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta}+s_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\
& + 4((2s_{2\beta}+s_{4\beta})c_{\alpha-\beta} + (c_{2\beta}+c_{4\beta})s_{\alpha-\beta})c_{1112}^T \\
& + 2((1-c_{4\beta})c_{\alpha-\beta} + s_{4\beta}s_{\alpha-\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\
& + 4((2s_{2\beta}-s_{4\beta})c_{\alpha-\beta} + (c_{2\beta}-c_{4\beta})s_{\alpha-\beta})c_{2221}^T \\
& \left. + ((3-4c_{2\beta}+c_{4\beta})c_{\alpha-\beta} + (2s_{2\beta}-s_{4\beta})s_{\alpha-\beta})c_{2222}^T \right), \tag{C.15}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ Z_L) = & \frac{t}{8f^2} \left( (3+4c_{2\beta}+c_{4\beta})c_{1111}^H + 4(2s_{2\beta}+s_{4\beta})c_{1112}^H \right. \\
& + 2(1-c_{4\beta})(c_{1122}^H + c_{1221}^H + c_{1212}^H) \\
& \left. + 4(2s_{2\beta}-s_{4\beta})c_{2221}^H + (3-4c_{2\beta}+c_{4\beta})c_{2222}^H \right) \tag{C.16}
\end{aligned}$$

$$= \mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ Z_L)_{\text{cust}}, \tag{C.17}$$

$$\begin{aligned}
\mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ W_L^+) = & -\frac{s}{8f^2} \left( (3+4c_{2\beta}+c_{4\beta})(c_{1111}^H + 3c_{1111}^T) + 4(2s_{2\beta}+s_{4\beta})(c_{1112}^H + 3c_{1112}^T) \right. \\
& + 2(1-c_{4\beta})(c_{1122}^H + c_{1221}^H + c_{1212}^H + 3(c_{1122}^T + c_{1221}^T + c_{1212}^T)) \\
& \left. + 4(2s_{2\beta}-s_{4\beta})(c_{2221}^H + 3c_{2221}^T) + (3-4c_{2\beta}+c_{4\beta})(c_{2222}^H + 3c_{2222}^T) \right) \tag{C.18}
\end{aligned}$$

$$\begin{aligned}
= & \mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ W_L^+)_{\text{cust}} \\
& - \frac{3s}{f^2} \left( (3+4c_{2\beta}+c_{4\beta})c_{1111}^T + 4(2s_{2\beta}+s_{4\beta})c_{1112}^T \right. \\
& + 2(1-c_{4\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\
& \left. + 4(2s_{2\beta}-s_{4\beta})c_{2221}^T + (3-4c_{2\beta}+c_{4\beta})c_{2222}^T \right). \tag{C.19}
\end{aligned}$$

Secondly, in the following amplitudes, one of the emitted particle is a heavy Higgs

boson,  $X_1 \in \{W_L^\pm, Z_L, h\}$  and  $X_2 \in \{H^\pm, A, H\}$ :

$$\begin{aligned} \mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ H^-) = & \frac{s+t}{8f^2} \left( -(2s_{2\beta} + s_{4\beta})(c_{1111}^H + 3c_{1111}^T) + 4(c_{2\beta} + c_{4\beta})(c_{1112}^H + 3c_{1112}^T) \right. \\ & + 2s_{4\beta}(c_{1122}^H + c_{1221}^H + c_{1212}^H + 3(c_{1122}^T + c_{1221}^T + c_{1212}^T)) \\ & \left. + 4(c_{2\beta} - c_{4\beta})(c_{2221}^H + 3c_{2221}^T) + (2s_{2\beta} - s_{4\beta})(c_{2222}^H + 3c_{2222}^T) \right) \end{aligned} \quad (\text{C.20})$$

$$\begin{aligned} = & \mathcal{M}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}} \\ & + \frac{3(s+t)}{8f^2} \left( -(2s_{2\beta} + s_{4\beta})c_{1111}^T + 4(c_{2\beta} + c_{4\beta})c_{1112}^T \right. \\ & + 2s_{4\beta}(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\ & \left. + 4(c_{2\beta} - c_{4\beta})c_{2221}^T + (2s_{2\beta} - s_{4\beta})c_{2222}^T \right), \end{aligned} \quad (\text{C.21})$$

$$\begin{aligned} \mathcal{M}(W_L^+ W_L^- \rightarrow hH) = & \frac{s}{8f^2} \left( -((2s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} + (1 + 2c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})c_{1111}^H \right. \\ & + 4((c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})c_{1112}^H \\ & + 2(s_{4\beta}c_{2(\alpha-\beta)} + (1 + c_{4\beta})s_{2(\alpha-\beta)})c_{1122}^H \\ & + 2(s_{4\beta}c_{2(\alpha-\beta)} - (1 - c_{4\beta})s_{2(\alpha-\beta)})(c_{1221}^H + c_{1212}^H) \\ & + 4((c_{2\beta} - c_{4\beta})c_{2(\alpha-\beta)} - (s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})c_{2221}^H \\ & \left. + ((2s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} - (1 - 2c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})c_{2222}^H \right) \end{aligned} \quad (\text{C.22})$$

$$= \mathcal{M}(W_L^+ W_L^- \rightarrow hH)_{\text{cust}}, \quad (\text{C.23})$$

$$\begin{aligned} \mathcal{M}(W_L^+ W_L^- \rightarrow hA) = & i \frac{s+2t}{8f^2} \left( -((2s_{2\beta} + s_{4\beta})c_{\alpha-\beta} + (1 - c_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\ & + 4((c_{2\beta} + c_{4\beta})c_{\alpha-\beta} - s_{4\beta}s_{\alpha-\beta})c_{1112}^T \\ & + 2(s_{4\beta}c_{\alpha-\beta} + (3 + c_{4\beta})s_{\alpha-\beta})c_{1122}^T \\ & + 2(s_{4\beta}c_{\alpha-\beta} - (1 - c_{4\beta})s_{\alpha-\beta})(c_{1221}^T + c_{1212}^T) \\ & + 4((c_{2\beta} - c_{4\beta})c_{\alpha-\beta} + s_{4\beta}s_{\alpha-\beta})c_{2221}^T \\ & \left. + ((2s_{2\beta} - s_{4\beta})c_{\alpha-\beta} + (1 - c_{4\beta})s_{\alpha-\beta})c_{2222}^T \right) \end{aligned} \quad (\text{C.24})$$

$$\begin{aligned} = & \mathcal{M}(W_L^+ W_L^- \rightarrow hA)_{\text{cust}} \\ & + i \frac{s+2t}{8f^2} \left( -((2s_{2\beta} + s_{4\beta})c_{\alpha-\beta} + (1 - c_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\ & + 4((c_{2\beta} + c_{4\beta})c_{\alpha-\beta} - s_{4\beta}s_{\alpha-\beta})c_{1112}^T \\ & + 2(s_{4\beta}c_{\alpha-\beta} + (3 + c_{4\beta})s_{\alpha-\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\ & + 4((c_{2\beta} - c_{4\beta})c_{\alpha-\beta} + s_{4\beta}s_{\alpha-\beta})c_{2221}^T \\ & \left. + ((2s_{2\beta} - s_{4\beta})c_{\alpha-\beta} + (1 - c_{4\beta})s_{\alpha-\beta})c_{2222}^T \right) \\ & - i \frac{s+2t}{3f^2} s_{\alpha-\beta} \left( 3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H) \right), \end{aligned} \quad (\text{C.25})$$

$$\begin{aligned}
\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L H) = & i \frac{s+2t}{8f^2} \left( ((2s_{2\beta} + s_{4\beta})c_{\alpha-\beta} + (3 + 4c_{2\beta} + c_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\
& - 4((c_{2\beta} + c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta} + s_{4\beta})s_{\alpha-\beta})c_{1112}^T \\
& - 2(s_{4\beta}c_{\alpha-\beta} - (1 - c_{4\beta})s_{\alpha-\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\
& - 4((c_{2\beta} - c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta} - s_{4\beta})s_{\alpha-\beta})c_{2221}^T \\
& \left. - ((2s_{2\beta} - s_{4\beta})c_{\alpha-\beta} - (3 - 4c_{2\beta} + c_{4\beta})s_{\alpha-\beta})c_{2222}^T \right), \quad (\text{C.26})
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}(W_L^+ W_L^- \rightarrow Z_L A) = & \frac{s}{8f^2} \left( -(2s_{2\beta} + s_{4\beta})c_{1111}^H + 4(c_{2\beta} + c_{4\beta})c_{1112}^H \right. \\
& + 2s_{4\beta}(c_{1122}^H + c_{1221}^H + c_{1212}^H) \\
& \left. + 4(c_{2\beta} - c_{4\beta})c_{2221}^H + (2s_{2\beta} - s_{4\beta})c_{2222}^H \right) \quad (\text{C.27})
\end{aligned}$$

$$= \mathcal{M}(W_L^+ W_L^- \rightarrow Z_L A)_{\text{cust}}, \quad (\text{C.28})$$

$$\begin{aligned}\mathcal{M}(Z_L Z_L \rightarrow W_L^+ H^-) &= \frac{s}{8f^2} \left( -(2s_{2\beta} + s_{4\beta})c_{1111}^H + 4(c_{2\beta} + c_{4\beta})c_{1112}^H \right. \\ &\quad + 2s_{4\beta}(c_{1122}^H + c_{1221}^H + c_{1212}^H) \\ &\quad \left. + 4(c_{2\beta} - c_{4\beta})c_{2221}^H + (2s_{2\beta} - s_{4\beta})c_{2222}^H \right) \end{aligned} \quad (\text{C.29})$$

$$= \mathcal{M}(Z_L Z_L \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (\text{C.30})$$

$$\begin{aligned}\mathcal{M}(Z_L Z_L \rightarrow hH) &= \frac{s}{8f^2} \left( -((2s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} + (1 + 2c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})(c_{1111}^H + 3c_{1111}^T) \right. \\ &\quad + 4((c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})(c_{1112}^H + 3c_{1112}^T) \\ &\quad + (2s_{4\beta}c_{2(\alpha-\beta)} - (1 - 2c_{4\beta})s_{2(\alpha-\beta)})(c_{1122}^H + c_{1221}^H + 3(c_{1122}^T + c_{1221}^T)) \\ &\quad + 3s_{2(\alpha-\beta)}(c_{1122}^H - c_{1221}^H - c_{1122}^T + c_{1221}^T) \\ &\quad + 2(s_{4\beta}c_{2(\alpha-\beta)} + c_{4\beta}s_{2(\alpha-\beta)})(c_{1212}^H + 3c_{1212}^T) \\ &\quad + 4((c_{2\beta} - c_{4\beta})c_{2(\alpha-\beta)} - (s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})(c_{2221}^H + 3c_{2221}^T) \\ &\quad \left. + ((2s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} - (1 - 2c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})(c_{2222}^H + 3c_{2222}^T) \right) \end{aligned} \quad (\text{C.31})$$

$$\begin{aligned}&= \mathcal{M}(Z_L Z_L \rightarrow hH)_{\text{cust}} \\ &\quad + \frac{3s}{8f^2} \left( -((2s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} + (1 + 2c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})c_{1111}^T \right. \\ &\quad + 4((c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})c_{1112}^T \\ &\quad + ((-1 + c_{4\beta})s_{2(\alpha-\beta)} + s_{4\beta}c_{2(\alpha-\beta)})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\ &\quad + 4((c_{2\beta} - c_{4\beta})c_{2(\alpha-\beta)} - (s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})c_{2221}^T \\ &\quad \left. + ((2s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} - (1 - 2c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})c_{2222}^T \right) \\ &\quad + \frac{s}{4f^2} s_{2(\alpha-\beta)} \left( 3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H) \right), \end{aligned} \quad (\text{C.32})$$

$$\mathcal{M}(Z_L Z_L \rightarrow hA) = 0, \quad (\text{C.33})$$

$$\mathcal{M}(Z_L Z_L \rightarrow Z_L H) = 0, \quad (\text{C.34})$$

$$\mathcal{M}(Z_L Z_L \rightarrow Z_L A) = 0. \quad (\text{C.35})$$



$$\begin{aligned}
\mathcal{M}(W_L^+ Z_L \rightarrow H^+ h) = & i \frac{t}{2f^2} s_{\alpha-\beta} (-c_{1221}^H + c_{1212}^H) \\
& + i \frac{2s+t}{8f^2} \left( ((2s_{2\beta} + s_{4\beta})c_{\alpha-\beta} - (1 - c_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\
& \quad - 4((c_{2\beta} + c_{4\beta})c_{\alpha-\beta} - s_{4\beta}s_{\alpha-\beta})c_{1112}^T \\
& \quad - 2(s_{4\beta}c_{\alpha-\beta} - (1 - c_{4\beta})s_{\alpha-\beta})c_{1122}^T \\
& \quad - 2(s_{4\beta}c_{\alpha-\beta} + (1 + c_{4\beta})s_{\alpha-\beta})(c_{1221}^T + c_{1212}^T) \\
& \quad - 4((c_{2\beta} - c_{4\beta})c_{\alpha-\beta} + s_{4\beta}s_{\alpha-\beta})c_{2221}^T \\
& \quad \left. - ((2s_{2\beta} - s_{4\beta})c_{\alpha-\beta} + (1 - c_{4\beta})s_{\alpha-\beta})c_{2222}^T \right) \quad (C.36)
\end{aligned}$$

$$\begin{aligned}
= & \mathcal{M}(W_L^+ Z_L \rightarrow H^+ h)_{\text{cust}} \\
& + i \frac{2s+t}{8f^2} \left( ((2s_{2\beta} + s_{4\beta})c_{\alpha-\beta} - (1 - c_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\
& \quad - 4((c_{2\beta} + c_{4\beta})c_{\alpha-\beta} - s_{4\beta}s_{\alpha-\beta})c_{1112}^T \\
& \quad - 2(s_{4\beta}c_{\alpha-\beta} - (1 - c_{4\beta})s_{\alpha-\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\
& \quad - 4((c_{2\beta} - c_{4\beta})c_{\alpha-\beta} + s_{4\beta}s_{\alpha-\beta})c_{2221}^T \\
& \quad \left. - ((2s_{2\beta} - s_{4\beta})c_{\alpha-\beta} + (1 - c_{4\beta})s_{\alpha-\beta})c_{2222}^T \right) \\
& - i \frac{2s+t}{6f^2} s_{\alpha-\beta} \left( 3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H) \right), \quad (C.37)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ H) = & i \frac{2s+t}{8f^2} \left( ((2s_{2\beta} + s_{4\beta})c_{\alpha-\beta} + (3 + 4c_{2\beta} + c_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\
& \quad - 4((c_{2\beta} + c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta} + s_{4\beta})s_{\alpha-\beta})c_{1112}^T \\
& \quad - 2(s_{4\beta}c_{\alpha-\beta} - (1 - c_{4\beta})s_{\alpha-\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\
& \quad - 4((c_{2\beta} - c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta} - s_{4\beta})s_{\alpha-\beta})c_{2221}^T \\
& \quad \left. - ((2s_{2\beta} - s_{4\beta})c_{\alpha-\beta} - (3 - 4c_{2\beta} + c_{4\beta})s_{\alpha-\beta})c_{2222}^T \right), \quad (C.38)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ A) = & \frac{t}{8f^2} \left( -(2s_{2\beta} + s_{4\beta})c_{1111}^H + 4(c_{2\beta} + c_{4\beta})c_{1112}^H \right. \\
& \quad + 2s_{4\beta}(c_{1122}^H + c_{1221}^H + c_{1212}^H) \\
& \quad \left. + 4(c_{2\beta} - c_{4\beta})c_{2221}^H + (2s_{2\beta} - s_{4\beta})c_{2222}^H \right) \quad (C.39)
\end{aligned}$$

$$= \mathcal{M}(W_L^+ Z_L \rightarrow W_L^+ A)_{\text{cust}}, \quad (C.40)$$

$$\begin{aligned}
\mathcal{M}(W_L^+ Z_L \rightarrow H^+ Z_L) = & \frac{t}{8f^2} \left( -(2s_{2\beta} + s_{4\beta})c_{1111}^H + 4(c_{2\beta} + c_{4\beta})c_{1112}^H \right. \\
& \quad + 2s_{4\beta}(c_{1122}^H + c_{1221}^H + c_{1212}^H) \\
& \quad \left. + 4(c_{2\beta} - c_{4\beta})c_{2221}^H + (2s_{2\beta} - s_{4\beta})c_{2222}^H \right) \quad (C.41)
\end{aligned}$$

$$= \mathcal{M}(W_L^+ Z_L \rightarrow H^+ Z_L)_{\text{cust}}, \quad (C.42)$$

$$\begin{aligned}
\mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ H^+) = & -\frac{s}{8f^2} \left( -(2s_{2\beta} + s_{4\beta})(c_{1111}^H + 3c_{1111}^T) + 4(c_{2\beta} + c_{4\beta})(c_{1112}^H + 3c_{1112}^T) \right. \\
& + 2s_{4\beta}(c_{1122}^H + c_{1221}^H + c_{1212}^H + 3(c_{1122}^T + c_{1221}^T + c_{1212}^T)) \\
& \left. + 4(c_{2\beta} - c_{4\beta})(c_{2221}^H + 3c_{2221}^T) + (2s_{2\beta} - s_{4\beta})(c_{2222}^H + 3c_{2222}^T) \right) \\
& \tag{C.43}
\end{aligned}$$

$$\begin{aligned}
= & \mathcal{M}(W_L^+ W_L^+ \rightarrow W_L^+ H^+)_{\text{cust}} \\
& - \frac{3s}{8f^2} \left( -(2s_{2\beta} + s_{4\beta})c_{1111}^T + 4(c_{2\beta} + c_{4\beta})c_{1112}^T \right. \\
& + 2s_{4\beta}(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\
& \left. + 4(c_{2\beta} - c_{4\beta})c_{2221}^T + (2s_{2\beta} - s_{4\beta})c_{2222}^T \right). \tag{C.44}
\end{aligned}$$

Finally, we show the amplitudes of double heavy Higgs boson productions i.e.  $X_1, X_2 \in$

$\{H^\pm, A, H\}$ :

$$\begin{aligned}\mathcal{M}(W_L^+ W_L^- \rightarrow H^+ H^-) &= \frac{s+t}{8f^2} \left( (1-c_{4\beta})(c_{1111}^H - 2c_{1212}^H + c_{2222}^H + 3(c_{1111}^T - 2c_{1212}^T + c_{2222}^T)) \right. \\ &\quad - 4s_{4\beta}(c_{1112}^H - c_{2221}^H + 3(c_{1112}^T - c_{2221}^T)) \\ &\quad \left. + 2(1+c_{4\beta})(c_{1122}^H + c_{1221}^H + 3(c_{1122}^T + c_{1221}^T)) \right) \\ &\quad + \frac{s-t}{2f^2}(c_{1122}^H - c_{1221}^H - c_{1122}^T + c_{1221}^T) \quad (\text{C.45})\end{aligned}$$

$$\begin{aligned}&= \mathcal{M}(W_L^+ W_L^- \rightarrow H^+ H^-)_{\text{cust}} \\ &\quad + \frac{s+t}{8f^2} \left( 3(1-c_{4\beta})(c_{1111}^T + c_{2222}^T) - 12s_{4\beta}(c_{1112}^T - c_{2221}^T) \right. \\ &\quad \left. + 2(s+5t+3(s+t)c_{4\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \right), \quad (\text{C.46})\end{aligned}$$

$$\begin{aligned}\mathcal{M}(W_L^+ W_L^- \rightarrow HH) &= \frac{s}{8f^2} \left( (2(1+c_{2\beta}) - (1+2c_{2\beta}+c_{4\beta})c_{2(\alpha-\beta)} + (2s_{2\beta}+s_{4\beta})s_{2(\alpha-\beta)})c_{1111}^H \right. \\ &\quad + 4(s_{2\beta} - (s_{2\beta}+s_{4\beta})c_{2(\alpha-\beta)} - (c_{2\beta}+c_{4\beta})s_{2(\alpha-\beta)})c_{1112}^H \\ &\quad + 2(2+(1+c_{4\beta})c_{2(\alpha-\beta)} - s_{4\beta}s_{2(\alpha-\beta)})c_{1122}^H \\ &\quad - 2((1-c_{4\beta})c_{2(\alpha-\beta)} + s_{4\beta}s_{2(\alpha-\beta)})(c_{1221}^H + c_{1212}^H) \\ &\quad + 4(s_{2\beta} - (s_{2\beta}-s_{4\beta})c_{2(\alpha-\beta)} - (c_{2\beta}-c_{4\beta})s_{2(\alpha-\beta)})c_{2221}^H \\ &\quad \left. + (2(1-c_{2\beta}) - (1-2c_{2\beta}+c_{4\beta})c_{2(\alpha-\beta)} - (2s_{2\beta}-s_{4\beta})s_{2(\alpha-\beta)})c_{2222}^H \right) \quad (\text{C.47})\end{aligned}$$

$$= \mathcal{M}(W_L^+ W_L^- \rightarrow HH)_{\text{cust}}, \quad (\text{C.48})$$

$$\begin{aligned}\mathcal{M}(W_L^+ W_L^- \rightarrow AA) &= \frac{s}{8f^2} \left( (1-c_{4\beta})(c_{1111}^H - 2c_{1221}^H - 2c_{1212}^H + c_{2222}^H) \right. \\ &\quad \left. - 4s_{4\beta}(c_{1112}^H - c_{2221}^H) + 2(3+c_{4\beta})c_{1122}^H \right) \quad (\text{C.49})\end{aligned}$$

$$= \mathcal{M}(W_L^+ W_L^- \rightarrow AA)_{\text{cust}}, \quad (\text{C.50})$$

$$\begin{aligned}\mathcal{M}(W_L^+ W_L^- \rightarrow HA) &= i \frac{s+2t}{8f^2} \left( ((1-c_{4\beta})c_{\alpha-\beta} + (2s_{2\beta}+s_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\ &\quad - 4(s_{4\beta}c_{\alpha-\beta} + (c_{2\beta}+c_{4\beta})s_{\alpha-\beta})c_{1112}^T \\ &\quad + 2((3+c_{4\beta})c_{\alpha-\beta} - s_{4\beta}s_{\alpha-\beta})c_{1122}^T \\ &\quad - 2((1-c_{4\beta})c_{\alpha-\beta} + s_{4\beta}s_{\alpha-\beta})(c_{1221}^T + c_{1212}^T) \\ &\quad + 4(s_{4\beta}c_{\alpha-\beta} - (c_{2\beta}-c_{4\beta})s_{\alpha-\beta})c_{2221}^T \\ &\quad \left. + ((1-c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta}-s_{4\beta})s_{\alpha-\beta})c_{2222}^T \right) \quad (\text{C.51})\end{aligned}$$

$$\begin{aligned}&= \mathcal{M}(W_L^+ W_L^- \rightarrow HA)_{\text{cust}} \\ &\quad + i \frac{s+2t}{8f^2} \left( ((1-c_{4\beta})c_{\alpha-\beta} + (2s_{2\beta}+s_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\ &\quad - 4(s_{4\beta}c_{\alpha-\beta} + (c_{2\beta}+c_{4\beta})s_{\alpha-\beta})c_{1112}^T \\ &\quad + 2((3+c_{4\beta})c_{\alpha-\beta} - s_{4\beta}s_{\alpha-\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\ &\quad + 4(s_{4\beta}c_{\alpha-\beta} - (c_{2\beta}-c_{4\beta})s_{\alpha-\beta})c_{2221}^T \\ &\quad \left. + ((1-c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta}-s_{4\beta})s_{\alpha-\beta})c_{2222}^T \right) \\ &\quad - i \frac{s+2t}{3f^2} c_{\alpha-\beta} \left( 3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H) \right), \quad (\text{C.52})\end{aligned}$$

$$\begin{aligned}\mathcal{M}(Z_L Z_L \rightarrow H^+ H^-) &= \frac{s}{8f^2} \left( (1 - c_{4\beta})(c_{1111}^H - 2c_{1221}^H - 2c_{1212}^H + c_{2222}^H) \right. \\ &\quad \left. - 4s_{4\beta}(c_{1112}^H - c_{2221}^H) + 2(3 + c_{4\beta})c_{1122}^H \right) \end{aligned} \quad (\text{C.53})$$

$$= \mathcal{M}(Z_L Z_L \rightarrow H^+ H^-)_{\text{cust}}, \quad (\text{C.54})$$

$$\begin{aligned}\mathcal{M}(Z_L Z_L \rightarrow HH) &= \frac{s}{8f^2} \left( (2(1 + c_{2\beta}) - (1 + 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} + (2s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})(c_{1111}^H + 3c_{1111}^T) \right. \\ &\quad + 4(s_{2\beta} - (s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} - (c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})(c_{1112}^H + 3c_{1112}^T) \\ &\quad + (1 - (1 - 2c_{4\beta})c_{2(\alpha-\beta)} - 2s_{4\beta}s_{2(\alpha-\beta)})(c_{1122}^H + c_{1221}^H + 3(c_{1122}^T + c_{1221}^T)) \\ &\quad + 3(1 + c_{2(\alpha-\beta)})(c_{1122}^H - c_{1221}^H - c_{1122}^T + c_{1221}^T) \\ &\quad + 2(1 + c_{4\beta}c_{2(\alpha-\beta)} - s_{4\beta}s_{2(\alpha-\beta)})(c_{1212}^H + 3c_{1212}^T) \\ &\quad + 4(s_{2\beta} - (s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} - (c_{2\beta} - c_{4\beta})s_{2(\alpha-\beta)})(c_{2221}^H + 3c_{2221}^T) \\ &\quad \left. + (2(1 - c_{2\beta}) - (1 - 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (2s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})(c_{2222}^H + 3c_{2222}^T) \right) \end{aligned} \quad (\text{C.55})$$

$$\begin{aligned} &= \mathcal{M}(Z_L Z_L \rightarrow HH)_{\text{cust}} \\ &\quad + \frac{3s}{8f^2} \left( (2(1 + c_{2\beta}) - (1 + 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} + (2s_{2\beta} + s_{4\beta})s_{2(\alpha-\beta)})c_{1111}^T \right. \\ &\quad + 4(s_{2\beta} - (s_{2\beta} + s_{4\beta})c_{2(\alpha-\beta)} - (c_{2\beta} + c_{4\beta})s_{2(\alpha-\beta)})c_{1112}^T \\ &\quad - 2((1 - c_{4\beta})c_{2(\alpha-\beta)} + s_{4\beta}s_{2(\alpha-\beta)})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\ &\quad + 4(s_{2\beta} - (s_{2\beta} - s_{4\beta})c_{2(\alpha-\beta)} - (c_{2\beta} - c_{4\beta})s_{2(\alpha-\beta)})c_{2221}^T \\ &\quad + (2(1 - c_{2\beta}) - (1 - 2c_{2\beta} + c_{4\beta})c_{2(\alpha-\beta)} - (2s_{2\beta} - s_{4\beta})s_{2(\alpha-\beta)})c_{2222}^T \\ &\quad \left. + \frac{s}{4f^2}(1 + c_{2(\alpha-\beta)})(3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H)) \right), \end{aligned} \quad (\text{C.56})$$

$$\mathcal{M}(Z_L Z_L \rightarrow AA) = \frac{s}{2f^2} \left( 2c_{1122}^H - c_{1221}^H + c_{1212}^H - 3(c_{1221}^T - c_{1212}^T) \right) \quad (\text{C.57})$$

$$= \mathcal{M}(Z_L Z_L \rightarrow AA)_{\text{cust}} + \frac{s}{2f^2} \left( 3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H) \right), \quad (\text{C.58})$$

$$\mathcal{M}(Z_L Z_L \rightarrow HA) = 0, \quad (\text{C.59})$$

$$\begin{aligned}
\mathcal{M}(W_L^+ Z_L \rightarrow H^+ H) = & i \frac{t}{2f^2} c_{\alpha-\beta} (-c_{1221}^H + c_{1212}^H) \\
& + i \frac{2s+t}{8f^2} \left( -((1-c_{4\beta})c_{\alpha-\beta} + (2s_{2\beta} + s_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\
& + 4(s_{4\beta}c_{\alpha-\beta} + (c_{2\beta} + c_{4\beta})s_{\alpha-\beta})c_{1112}^T \\
& + 2((1-c_{4\beta})c_{\alpha-\beta} + s_{4\beta}s_{\alpha-\beta})c_{1122}^T \\
& - 2((1+c_{4\beta})c_{\alpha-\beta} - s_{4\beta}s_{\alpha-\beta})(c_{1221}^T + c_{1212}^T) \\
& - 4(s_{4\beta}c_{\alpha-\beta} - (c_{2\beta} - c_{4\beta})s_{\alpha-\beta})c_{2221}^T \\
& \left. - ((1-c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta} - s_{4\beta})s_{\alpha-\beta})c_{2222}^T \right) \quad (C.60)
\end{aligned}$$

$$\begin{aligned}
= & \mathcal{M}(W_L^+ Z_L \rightarrow H^+ H)_{\text{cust}} \\
& + i \frac{2s+t}{8f^2} \left( -((1-c_{4\beta})c_{\alpha-\beta} + (2s_{2\beta} + s_{4\beta})s_{\alpha-\beta})c_{1111}^T \right. \\
& + 4(s_{4\beta}c_{\alpha-\beta} + (c_{2\beta} + c_{4\beta})s_{\alpha-\beta})c_{1112}^T \\
& + 2((1-c_{4\beta})c_{\alpha-\beta} + s_{4\beta}s_{\alpha-\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \\
& - 4(s_{4\beta}c_{\alpha-\beta} - (c_{2\beta} - c_{4\beta})s_{\alpha-\beta})c_{2221}^T \\
& \left. - ((1-c_{4\beta})c_{\alpha-\beta} - (2s_{2\beta} - s_{4\beta})s_{\alpha-\beta})c_{2222}^T \right) \\
& + i \frac{s+2t}{3f^2} c_{\alpha-\beta} \left( 3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H) \right), \quad (C.61)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}(W_L^+ Z_L \rightarrow H^+ A) = & \frac{t}{8f^2} \left( (1-c_{4\beta})(c_{1111}^H - 2c_{1122}^H + c_{2222}^H) - 4s_{4\beta}(c_{1112}^H - c_{2221}^H) \right. \\
& \left. + 2(1+c_{4\beta})(c_{1221}^H + c_{1212}^H) \right) \\
& + \frac{2s+t}{2f^2} (c_{1221}^T - c_{1212}^T) \quad (C.62)
\end{aligned}$$

$$= \mathcal{M}(W_L^+ Z_L \rightarrow H^+ A)_{\text{cust}} - \frac{2s+t}{6f^2} \left( 3(c_{1221}^T + c_{1212}^T) - (c_{1221}^H - c_{1212}^H) \right), \quad (C.63)$$

$$\begin{aligned}
\mathcal{M}(W_L^+ W_L^+ \rightarrow H^+ H^+) = & \frac{s}{8f^2} \left( (1-c_{4\beta})(-c_{1111}^H + 2c_{1122}^H + 2c_{1221}^H - c_{2222}^H) \right. \\
& + 3(-c_{1111}^T + 2c_{1122}^T + 2c_{1221}^T - c_{2222}^T) \\
& + 4s_{4\beta}(c_{1112}^H - c_{2221}^H + 3(c_{1112}^T - c_{2221}^T)) \\
& \left. - 2(3+c_{4\beta})(c_{1212}^H + 3c_{1212}^T) \right) \quad (C.64)
\end{aligned}$$

$$\begin{aligned}
= & \mathcal{M}(W_L^+ W_L^+ \rightarrow H^+ H^+)_{\text{cust}} \\
& + \frac{3s}{8f^2} \left( (1-c_{4\beta})(-c_{1111}^T - c_{2222}^T) + 4s_{4\beta}(c_{1112}^T - c_{2221}^T) \right. \\
& \left. + 2(1-c_{4\beta})(c_{1122}^T + c_{1221}^T + c_{1212}^T) \right). \quad (C.65)
\end{aligned}$$

## D Elimination of the $\mathcal{O}^r$ and $\mathcal{O}^{HT}$

There are four kinds of operators,  $\mathcal{O}^H$ ,  $\mathcal{O}^r$ ,  $\mathcal{O}^T$  and  $\mathcal{O}^{HT}$  in the dimension-six derivative interactions of the NHD.  $\mathcal{O}^H$ ,  $\mathcal{O}^r$ ,  $\mathcal{O}^T$  and  $\mathcal{O}^{HT}$ . The operators,  $\mathcal{O}^r$  and  $\mathcal{O}^{HT}$ , can be eliminated by field redefinition. We consider the following redefinition:

$$H_i \rightarrow H_i + \frac{a_{ijkl}}{f^2} H_l (H_j^\dagger H_k), \quad (\text{D.1})$$

where  $a_{ijkl}$  are complex numbers. By the field redefinition on the kinetic term, we introduce the following terms:

$$(\partial_\mu H_i)^\dagger (\partial^\mu H_i) \rightarrow (\partial_\mu H_i)^\dagger (\partial^\mu H_i) + \left( \frac{a_{ijkl}}{f^2} (\partial_\mu H_i)^\dagger \partial^\mu (H_l (H_j^\dagger H_k)) + \text{H.c.} \right) + \mathcal{O}((H/f)^4), \quad (\text{D.2})$$

where indices  $i, j, k$  and  $l$  are summed over the species of the Higgs doublets. The second and the third terms can be written using  $\mathcal{O}^H$ ,  $\mathcal{O}^r$  and  $\mathcal{O}^{HT}$ :

$$\frac{a_{ijkl}}{f^2} \left( \partial_\mu H_i^\dagger \partial^\mu H_l (H_j^\dagger H_k) + \partial_\mu H_i^\dagger H_l \partial^\mu (H_j^\dagger H_k) \right) + \text{H.c.} \quad (\text{D.3})$$

$$= \frac{a_{ijkl}}{f^2} \left( O_{jkil}^r + \frac{1}{2} (O_{jkil}^H - O_{jkil}^{HT}) \right) + \frac{a_{ijkl}^*}{f^2} \left( O_{kjli}^r + \frac{1}{2} (O_{kjli}^H + O_{kjli}^{HT}) \right) \quad (\text{D.4})$$

$$= \frac{a_{kijl} + a_{ljik}^*}{f^2} O_{ijkl}^r + \frac{a_{kijl} + a_{ljik}^*}{2f^2} O_{ijkl}^H + \frac{-a_{kijl} + a_{ljik}^*}{2f^2} O_{ijkl}^{HT}. \quad (\text{D.5})$$

Accordingly, we can choose the conditions so as to eliminate  $\mathcal{O}^r$  and  $\mathcal{O}^{HT}$ :

$$\begin{aligned} a_{kijl} + a_{ljik}^* &= -\lambda_{ijkl}^r, \\ -a_{kijl} + a_{ljik}^* &= -\lambda_{ijkl}^{HT}. \end{aligned} \quad (\text{D.6})$$

It is clear that the elimination of  $\mathcal{O}^r$  affects the coefficients of  $\mathcal{O}^H$ . On the other hand, no contribution of  $\mathcal{O}^{HT}$  arises in the derivative interactions by the prescription.

The same result can be derived using the equation of motion. We can rewrite  $O_{ijkl}^r$  and  $O_{ijkl}^{HT}$  as

$$O_{ijkl}^r = \frac{1}{2} \partial_\mu (H_i^\dagger H_j \partial^\mu (H_k^\dagger H_l)) - \frac{1}{2} O_{ijkl}^H - \frac{1}{2} \left( H_i^\dagger H_j (H_k^\dagger \partial^2 H_l + (\partial^2 H_k)^\dagger H_l) \right), \quad (\text{D.7})$$

$$O_{ijkl}^{HT} = -H_i^\dagger H_j (H_k^\dagger \partial^2 H_l + (\partial^2 H_k)^\dagger H_l). \quad (\text{D.8})$$

Ignoring total derivative terms and using the equation of motion,  $\mathcal{O}^r$  can be written in terms of  $\mathcal{O}^H$ , whereas  $\mathcal{O}^{HT}$  does not contribute to the derivative interactions.

## E Cross sections of the central region

The cross section limited in the region of  $-1/2 < \cos \theta < 1/2$  is

$$\sigma_{1/2} = \frac{s}{32\pi f^4} \left( \frac{(2C_s - C_t)^2}{4} + \frac{C_t^2}{48} \right) \quad (\text{E.1})$$

for the amplitude given by Eq. (4.78). If same particles are included in the final state, the above cross section should be divided by two.

With the formula, cross sections are

$$\begin{aligned}\sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{13}{48} C_1(\beta)^2 \\ &= \frac{13}{32} \sigma(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}},\end{aligned}\quad (\text{E.2})$$

$$\begin{aligned}\sigma_{1/2}(W_L^+ W_L^- \rightarrow hh)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{1}{2} C_2(\alpha, \beta)^2 \\ &= \frac{1}{2} \sigma(W_L^+ W_L^- \rightarrow hh)_{\text{cust}},\end{aligned}\quad (\text{E.3})$$

$$\sigma_{1/2}(W_L^+ W_L^- \rightarrow Z_L Z_L)_{\text{cust}} = \frac{24}{13} \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (\text{E.4})$$

$$\sigma_{1/2}(Z_L Z_L \rightarrow W_L^+ W_L^-)_{\text{cust}} = \frac{48}{13} \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (\text{E.5})$$

$$\sigma_{1/2}(Z_L Z_L \rightarrow hh)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow hh)_{\text{cust}}, \quad (\text{E.6})$$

$$\sigma_{1/2}(W_L^+ Z_L \rightarrow W_L^+ Z_L)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (\text{E.7})$$

$$\sigma_{1/2}(W_L^+ W_L^+ \rightarrow W_L^+ W_L^+)_{\text{cust}} = \frac{24}{13} \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)_{\text{cust}}, \quad (\text{E.8})$$

$$\begin{aligned}\sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{13}{48} C_3(\beta)^2 \\ &= \frac{13}{32} \sigma(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}},\end{aligned}\quad (\text{E.9})$$

$$\begin{aligned}\sigma_{1/2}(W_L^+ W_L^- \rightarrow hH)_{\text{cust}} &= \frac{s}{32\pi f^4} C_4(\alpha, \beta)^2 \\ &= \frac{1}{2} \sigma(W_L^+ W_L^- \rightarrow hH)_{\text{cust}},\end{aligned}\quad (\text{E.10})$$

$$\begin{aligned}\sigma_{1/2}(W_L^+ W_L^- \rightarrow hA)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{1}{108} \sin^2(\alpha - \beta) (c_{1221}^H - c_{1212}^H)^2 \\ &= \frac{1}{8} \sigma(W_L^+ W_L^- \rightarrow hA)_{\text{cust}},\end{aligned}\quad (\text{E.11})$$

$$\sigma_{1/2}(W_L^+ W_L^- \rightarrow Z_L A)_{\text{cust}} = \frac{48}{13} \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (\text{E.12})$$

$$\sigma_{1/2}(Z_L Z_L \rightarrow W_L^+ H^-)_{\text{cust}} = \frac{48}{13} \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (\text{E.13})$$

$$\sigma_{1/2}(Z_L Z_L \rightarrow hH)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow hH)_{\text{cust}}, \quad (\text{E.14})$$

$$\sigma_{1/2}(W_L^+ Z_L \rightarrow H^+ h)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow hA)_{\text{cust}}, \quad (\text{E.15})$$

$$\sigma_{1/2}(W_L^+ Z_L \rightarrow W_L^+ A)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (\text{E.16})$$

$$\sigma_{1/2}(W_L^+ Z_L \rightarrow H^+ Z_L)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (\text{E.17})$$

$$\sigma_{1/2}(W_L^+ W_L^+ \rightarrow W_L^+ H^+)_{\text{cust}} = \frac{48}{13} \sigma_{1/2}(W_L^+ W_L^- \rightarrow W_L^+ H^-)_{\text{cust}}, \quad (\text{E.18})$$

$$\sigma_{1/2}(W_L^+ W_L^- \rightarrow H^+ H^-)_{\text{cust}} = \frac{s}{32\pi f^4} \frac{13}{48} \left( C_5(\beta)^2 - \frac{22}{13} C_5(\beta) (c_{1221}^H - c_{1122}^H) + (c_{1221}^H - c_{1122}^H)^2 \right), \quad (\text{E.19})$$

$$\begin{aligned} \sigma_{1/2}(W_L^+ W_L^- \rightarrow HH)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{1}{2} C_6(\alpha, \beta)^2 \\ &= \frac{1}{2} \sigma_{1/2}(W_L^+ W_L^- \rightarrow HH)_{\text{cust}}, \end{aligned} \quad (\text{E.20})$$

$$\begin{aligned} \sigma_{1/2}(W_L^+ W_L^- \rightarrow AA)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{1}{2} C_5(\beta)^2 \\ &= \frac{1}{2} \sigma(W_L^+ W_L^- \rightarrow AA)_{\text{cust}}, \end{aligned} \quad (\text{E.21})$$

$$\begin{aligned} \sigma_{1/2}(W_L^+ W_L^- \rightarrow HA)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{1}{108} \cos^2(\alpha - \beta) (c_{1221}^H - c_{1212}^H)^2 \\ &= \frac{1}{8} \sigma(W_L^+ W_L^- \rightarrow HA)_{\text{cust}}, \end{aligned} \quad (\text{E.22})$$

$$\sigma_{1/2}(Z_L Z_L \rightarrow H^+ H^-)_{\text{cust}} = 2\sigma_{1/2}(W_L^+ W_L^- \rightarrow AA)_{\text{cust}}, \quad (\text{E.23})$$

$$\sigma_{1/2}(Z_L Z_L \rightarrow HH)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow HH)_{\text{cust}}, \quad (\text{E.24})$$

$$\begin{aligned} \sigma_{1/2}(Z_L Z_L \rightarrow AA)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{1}{2} (c_{1122}^H - 3c_{1221}^T)^2 \\ &= \frac{1}{2} \sigma(Z_L Z_L \rightarrow AA)_{\text{cust}}, \end{aligned} \quad (\text{E.25})$$

$$\sigma_{1/2}(W_L^+ Z_L \rightarrow H^+ H)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow HA)_{\text{cust}}, \quad (\text{E.26})$$

$$\begin{aligned} \sigma_{1/2}(W_L^+ Z_L \rightarrow H^+ A)_{\text{cust}} &= \frac{s}{32\pi f^4} \frac{1}{4} \left( (C_5(\beta) - c_{1122}^H + c_{1221}^H - 3c_{1221}^T)^2 \right. \\ &\quad \left. + \frac{1}{12} \left( C_5(\beta) + \frac{c_{1221}^H + 2c_{1212}^H}{3} - c_{1122}^H + c_{1221}^T \right)^2 \right), \end{aligned} \quad (\text{E.27})$$

$$\sigma_{1/2}(W_L^+ W_L^+ \rightarrow H^+ H^+)_{\text{cust}} = \sigma_{1/2}(W_L^+ W_L^- \rightarrow AA)_{\text{cust}}. \quad (\text{E.28})$$

## References

- [1] A. Nisati, for ATLAS collaboration, talk presented at Lepton-Photon Conference 2011, Mumbai, India, August 2011; V. Sharma, for CMS collaboration, talk presented at Lepton-Photon Conference 2011, Mumbai, India, August 2011; M. Verzocchi, for Tevatron collaboration, talk presented at Lepton-Photon Conference 2011, Mumbai, India, August 2011.
- [2] G. F. Giudice, C. Grojean, A. Pomarol, R. Rattazzi, JHEP **0706**, 045 (2007).
- [3] N. Arkani-Hamed, A. G. Cohen, H. Georgi, Phys. Lett. **B513**, 232-240 (2001);  
N. Arkani-Hamed, A. G. Cohen, T. Gregoire, J. G. Wacker, JHEP **0208**, 020 (2002).
- [4] R. Contino, Y. Nomura, A. Pomarol, Nucl. Phys. **B671**, 148-174 (2003); K. Agashe,  
R. Contino, A. Pomarol, Nucl. Phys. **B719**, 165-187 (2005).
- [5] I. Low, R. Rattazzi, A. Vichi, JHEP **1004**, 126 (2010).



- [6] S. R. Coleman, J. Wess, B. Zumino, Phys. Rev. **177**, 2239-2247 (1969); C. G. Callan, Jr., S. R. Coleman, J. Wess, B. Zumino, Phys. Rev. **177**, 2247-2250 (1969).
- [7] M. S. Chanowitz, M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985).
- [8] T. Han, D. Krohn, L. -T. Wang, W. Zhu, JHEP **1003**, 082 (2010).
- [9] R. Contino, C. Grojean, M. Moretti, F. Piccinini, R. Rattazzi, JHEP **1005**, 089 (2010).
- [10] R. Grober, M. Muhlleitner, JHEP **1106**, 020 (2011).
- [11] R. Contino, D. Marzocca, D. Pappadopulo and R. Rattazzi, JHEP **1110**, 081 (2011).
- [12] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire, J. G. Wacker, JHEP **0208**, 021 (2002); D. E. Kaplan, M. Schmaltz, JHEP **0310**, 039 (2003); W. Skiba, J. Terning, Phys. Rev. **D68**, 075001 (2003); B. Gripaios, A. Pomarol, F. Riva, J. Serra, JHEP **0904**, 070 (2009); M. Redi, B. Gripaios, JHEP **1008**, 116 (2010); M. Schmaltz, D. Stolarski, J. Thaler, JHEP **1009**, 018 (2010).
- [13] J. Mrazek, A. Pomarol, R. Rattazzi, M. Redi, J. Serra, A. Wulzer, Nucl. Phys. **B853**, 1-48 (2011).
- [14] B. Grzadkowski, M. Maniatis and J. Wudka, JHEP **1111**, 030 (2011); C. C. Nishi, Phys. Rev. **D83**, 095005 (2011).
- [15] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, (Addison-Wesley, Redwood City, CA, 1990).
- [16] H. E. Haber, arXiv:hep-ph/9501320.